Theoretical Economics 6 (2011), 185–217 1555-7561/20110185

Manipulative auction design

Philippe Jehiel
Department of Economics, Paris School of Economics
and Department of Economics, University College London

This paper considers an auction design framework in which bidders get partial feedback about the distribution of bids submitted in earlier auctions: either bidders are asymmetric but past bids are disclosed in an anonymous way or several auction formats are being used and the distribution of bids, but not the associated formats, is disclosed. I employ the analogy-based expectation equilibrium (Jehiel 2005) to model such situations. First-price auctions in which past bids are disclosed in an anonymous way generate more revenues than second-price auctions while achieving an efficient outcome in the asymmetric private values two-bidder case with independent distributions. Besides, by using several auction formats with coarse feedback, a designer can always extract more revenues than in Myerson's optimal auction, and yet less revenues than in the full information case whenever bidders enjoy ex post quitting rights and the assignment and payment rules are monotonic in bids. These results suggest an important role of feedback disclosure as a novel instrument in mechanism design.

Keywords. Auction design, analogy-based expectation equilibrium, manipulation.

JEL classification. C72, D82, D84.

1. Introduction

Standard equilibrium approaches of games with incomplete information (à la Harsanyi) assume that players know the distributions of signals held by other players as well as those players' strategies as a function of their signals (see Harsanyi 1995). Yet, this requires a lot of knowledge that need not be easily accessible to players. Modern approaches to equilibrium rely on learning to justify this knowledge (see Fudenberg and Levine 1998 for an overview of the literature on learning in games). But, it is in general questionable that enough information feedback is available to the players at the learning stage for convergence to equilibrium to be reasonably expected.

Philippe Jehiel: jehiel@enpc.fr
I would like to thank B. Lipman (the editor) and two anonymous referees for helpful comments. I am also grateful to I. Esponda, L. Lamy, M. Pagnozzi, R. Spiegler, B. Wilson, J. Wooders, A. Wolinsky, and seminar participants at Roy seminar (Paris), Northwestern University, the Conference in Honor of Ehud Kalai (Jerusalem, 2007), the CEMMAP Conference on Auctions (UCL, 2008), the Workshop on Biased Beliefs (UCL, 2008), SWET (Madrid, 2008), and the Third World Congress of Game Theory for useful Comments. I thank the European Research Council for support.

DOI: 10.3982/TE687

For the sake of illustration, consider a series of first-price auctions of similar objects (say, iPhones) involving each time new bidders of observable characteristics $i = 1, 2, \ldots, n$ (say, bidders of different age). Assume that every bidder knows his own valuation for the object but not that of other bidders, and assume that the distribution of valuation depends on the observable characteristic i. In many contexts, it seems unlikely that bidders would a priori know these distributions and how they depend on i. In such cases, bidders would look at the bids submitted in earlier similar auctions so as to form a judgement as to what the distribution of others' bids is likely to be in the current auction of interest.[1] In many practical auction designs such as those used on eBay or treasury auctions, bidders have access only to the aggregate distribution of bids in past auctions without being informed of the characteristics of the bidders of the corresponding bids. It is then dubious that bidders would be able to play a best response to the actual distribution of bids of the other bidders because there is no way a bidder can assess the distribution of bids conditional on the observable characteristic based on the feedback he receives. Instead, bidders are more likely to play a best response to the conjecture that all bidders—no matter what their characteristic is—bid according to the aggregate distribution of bids that mix the distribution of bids of all bidders. In the long run, assuming convergence of the overall process, bidders are not playing a Nash equilibrium, but an analogy-based expectation equilibrium with bidder-anonymous analogy partition (see Jehiel 2005 for the first exposition of this concept and Section 2 for an application to the private value auction setup considered here).[2]

As an alternative example, consider promotions in an organization. Each promotion takes the form of a contest between several employees who each make a proposal for the job task specification in case of promotion (i.e., what they will effectively do for the organization if promoted). Each employee has some private interest for the promotion (depending on how he values power, social status, responsibilities…), which is known to him only. Besides, while the criterion for the current promotion (based on the proposals) is typically known to the contestants, the criterion may typically change from one promotion to the next.[3] Nash equilibrium would require that contestants know the distribution of others' proposals given the criterion applicable to the current promotion. Assuming that contestants form their expectations by looking at past promotions, this would require that employees have access to the joint distribution of proposals and corresponding criterion. Yet, if employees have access only to the distribution of proposals (and not of the corresponding criterion), contestants are not able to play a Nash equilibrium. Instead, I propose that contestants play a best response (given the current criterion) to the conjecture that the distribution of proposals is the same irrespective of the

[1]Typically, on eBay, one has access to the history of bids made in auctions of similar objects run in the last month.

[2]In the language of econometrics, the model is not identifiable. Most applied econometricians would particularize the model by making an extra symmetry assumption (see Athey and Haile 2006). In a sense, this paper is assuming that bidders follow the same line as the econometricians, even though this does not boil down to assuming that the true underlying problem is symmetric, as we shall see.

[3]In the formal model to be described in Section 2, the private interest should be identified with the valuation, and the proposal (even though generally multidimensional) should be identified with the bid (maybe as in scoring auctions).

criterion (and that it corresponds to the aggregate distribution of proposals they have access to). This is again an analogy-based expectation equilibrium with appropriately chosen analogy partitions.

In this paper, I generalize the above two examples. I consider one-object private values auction environments in which the valuations are independently distributed across bidders, bidders receive coarse feedback about the distribution of previous bids, and every single bidder participates in just one auction. Specifically, I consider situations in which, as in the first-price auction example, only the aggregate distribution of bids with no reference to the characteristics of bidders is disclosed, and also situations in which, as in the promotion example, different auction formats are being used (this is the analog of different criteria being used) and only the aggregate distribution of bids across the various auction formats is being disclosed. Any combination of such coarse disclosures is also allowed. The equilibrium concept that is used to describe the interaction of bidders with such coarse feedback disclosure—the analogy-based expectation equilibrium—requires that bidders play a best response to the aggregate distribution of bids, as given by the feedback they receive.

I explore whether and when it is the case that the designer is better off when bidders play an analogy-based expectation equilibrium rather than a Nash equilibrium and how the answer is affected by the specific forms of feedback disclosure and auction rules. Addressing such questions opens new avenues in mechanism design. As it turns out, providing coarse feedback about past behaviors—as considered in this paper—can enhance the designer's objective, which suggests the relevance of an instrument not previously considered in mechanism design.[4]

To highlight the potential role of feedback disclosure in mechanism design, I assume that (in addition to the format(s)) the designer is free to choose which kind of feedback (within the class specified above) to disclose to bidders. In the promotion example, choosing which feedback to disclose sounds natural for the designer given that the feedback is, in principle, under the control of the organization. In the auction example, this is an idealization that would fit well if a single seller had many similar objects to auction off over time. My framework in the auction case can more usefully be interpreted as providing insights as to what kind of feedback policy an auction house such as eBay should adopt so as to increase the revenues of sellers (which in turn determine the revenues of the auction house through the fees).

I first assume that the main objective of the designer is welfare maximization whereas the auxiliary goal is revenue (as is the case in many government auctions). The main question I address with this objective in mind is, "Can the designer do better than using a second-price auction (or equivalently an ascending-price auction) by using a coarse feedback device?"

In the classic setup (relying on Nash equilibrium), the so-called revenue equivalence theorem implies that the designer can do no better. This is so because the second-price auction induces an efficient outcome and any efficient mechanism that respects the

[4] For a related investigation in the context of symmetric first-price auctions with affiliated signals, see the independent work of Esponda (2008a), which is discussed in Section 5.1.

participation constraints of bidders must achieve a revenue no greater than that of the second-price auction (see, for example, Milgrom 2004 for an exposition of the revenue equivalence theorem).

In my setup, I show that the designer can sometimes do better, thereby illustrating a failure of the revenue equivalence theorem when the solution concept is the analogy-based expectation equilibrium. Specifically, in the case of two bidders with asymmetric distributions of valuations, I show that the first-price auction in which the designer provides as the aggregate distribution of bids feedback, with no reference to the characteristic of the bidders (as considered above), always induces an efficient outcome and always generates an expected revenue that is strictly greater than that of the second-price auction no matter what the distributions of valuations are. I also provide conditions ensuring that such a result holds true when there are more than two bidders.[5]

I next consider the case in which the designer is only interested in revenues and I assume that bidders can always veto the transaction ex post (thereby limiting the scope for manipulation). The main question I am interested in is, "Can the designer generate more revenues than in the classic optimal auction characterized by Myerson (1981) and Riley and Samuelson (1981)?"

I show that this is always so and that the best revenue so obtained lies strictly below the full information maximal revenue. In other words, a clever use of coarse feedback coupled with a judicious choice of format(s) may reduce the informational rent left to bidders even though some rent must be left to bidders.[6]

The rest of the paper is organized as follows. In Section 2, the model is introduced together with the analogy-based expectation equilibrium and the mechanism design problem. In Section 3, the first-price auction with bidder-anonymous feedback partition is analyzed. In Section 4, it is shown how one can generate more revenues than in Myerson's optimal auction. Section 5 offers a discussion, including how the paper relates to the literature and what would happen if the transaction need not be approved ex post or if the designer could use shill bidders. Section 6 concludes. All proofs are given in the Appendix.

2. Basic definitions

An object is to be auctioned off and there are n bidders $i = 1, \dots, n$. Each bidder i knows his own valuation v_i for the object, but not that of the other bidders $j \neq i$. The distribution of valuations is independent across bidders. The valuation v_i is drawn from a distribution with support $[c, d]$ and (continuous) density $f_i(\cdot)$, where $f_i(v) > 0$ for all $v \in [c, d]$ and $d > c \geq 0$. Bidders have quasilinear preferences and they are risk neutral. That is, if a bidder with valuation v expects to win the object with probability p and expects to make

[5]Such clear-cut revenue comparisons (at least in the two-bidder case) should be contrasted with the ambiguous revenue ranking between the first-price auction and the second-price auction obtained with Nash equilibrium when bidders are asymmetric (see Maskin and Riley 2000).

[6]The auction design and feedback policy used to prove that one can go strictly beyond Myerson's optimal auction revenue require the use of several (in fact two) auction formats and that bidders be informed only of the aggregate distribution of bids over the various formats. Such auction designs do not clearly resemble existing ones, and more work is required to map this theoretical result to practical auction design. Such a construction may possibly shed light on why transparency need not be optimal in promotion contexts.

(an expected) transfer t to the seller, his expected utility is $pv - t$. The seller's valuation is v_s.

The object is auctioned off through possibly different auction formats M_k, $k \in K = \{1, \dots, r\}$, where format M_k is used with probability λ_k. Each auction format M_k (which together with λ_k is chosen by the designer) is restricted to take the following form.

- Bidders $i = 1, \dots, n$ simultaneously submit a bid $b_i \in [0, d]$.
- Based on the profile of bids $b = (b_i)_{i=1}^{i=n}$, with probability $\varphi_i^k(b)$ where $\sum_i \varphi_i^k(b) \leq 1$, bidder i is offered to obtain the object in exchange for a payment $\tau_i^k(b)$ to the seller. The transaction takes place if after being informed of $\tau_i^k(b)$, bidder i approves the terms of the contract.
- For every i, k, $\varphi_i^k(b)$ is a nondecreasing function of b_i and a nonincreasing function of b_j, $j \neq i$; $\tau_i^k(b)$ is a nondecreasing functions of b_i and of b_j, $j \neq i$. Moreover, if $b_i < v_s$, then $\varphi_i^k(b) = 0$.

The auction formats considered in this paper require that the bidders approve the terms of the contract ex post. That is, assuming that bidder i with valuation v_i is declared the winner with a tentative transaction price $\tau_i^k(b)$, there is effectively a transaction (at price $\tau_i^k(b)$) only if $v_i > \tau_i^k(b)$, and otherwise there is no transaction and the seller keeps the object.[7]

The monotonicity assumptions made on $\varphi_i^k(b)$ and $\tau_i^k(b)$ are in line with the auction interpretation: As a bidder increases his bid, his probability of winning (weakly) increases and so does the price this bidder must pay. As a competing bidder increases his bid, the probability of winning (weakly) decreases and the price paid in case of transaction (weakly) increases. Moreover, the requirement that if $b_i < v_s$, then $\varphi_i^k(b) = 0$ is meant to represent the idea that the valuation v_s of the seller is the minimum starting point of the auction (as the seller would refuse to sell at a price lower than v_s).

It should be mentioned that the first-price auction and the second-price auction with reserve price no smaller than v_s both belong to the above class of auctions, and that it is always possible to pick an auction in this class that achieves Myerson's optimal revenue no matter what the densities $f_i(\cdot)$ are. Moreover, the requirement that bidders should approve the terms of the contract ex post ensures that a bidder, no matter what bidding strategy he employs and no matter what he expects the distribution of other bids to be, is better off participating in the auction rather than staying outside.

I have in mind situations in which bidders have no prior idea about what the densities of valuations $f_i(\cdot)$, $i \in I$, are and in which every individual bidder participates in just one auction. To form their expectations about the distribution of other bids, bidders rely on the feedback about past play that is made available by the designer.

Specifically, when auction format M_k prevails, bidder i is assumed to be informed of the functions $\varphi_i^k(b)$ and $\tau_i^k(b)$ that apply to him in this format (but he need not be informed of other characteristics of M_k; see more on this below). Thus, if bidder i with

[7] I also assume that there is a tiny (rather than exactly zero) cost to cancelling out the transaction. This allows me to rule out crazy behaviors from bidders with valuations $v < v_s$.

valuation v_i bids b_i and expects the bid profile $b_{-i} = (b_j)_{j \neq i}$ to be distributed according to the random variable $\widetilde{b}_{-i}$, his perceived expected utility in M_k is

$$u_i^k(v_i, b_i; \widetilde{b}_{-i}) = \underset{\widetilde{b}_{-i}}{E}\left[\varphi_i^k(b_i, b_{-i}) \max(v_i - \tau_i^k(b_i, b_{-i}), 0)\right].$$

A strategy of bidder i is a family of bid functions $\beta_i = (\beta_i^k)_k$, one for each auction format M_k, where $\beta_i^k(v_i)$ denotes bidder i's bid in format M_k when i's valuation is v_i.[8]

Nash equilibrium would require that for each k and v_i, bidder i plays a best response to the *actual* distribution of bids of bidders $j \neq i$ in M_k. That is,

$$\beta_i^k(v_i) \in \arg\max_{b_i} u_i^k(v_i, b_i; \beta_{-i}^k),$$

where (with some slight abuse of notation) β_{-i}^k stands for the random variable of bids $(\beta_j^k(v_j))_{j \neq i}$ as generated by the densities $(f_j(\cdot))_{j \neq i}$.

As already highlighted, bidders are not assumed to know (or have access to) β_j^k for every j and k. Instead, each bidder i receives partial feedback about the distribution of bids observed in past auctions. They play a best response to this feedback (in a sense to be defined next) and a steady state is assumed to have been reached.

Specifically, I consider the following class of partial feedback. Endow each bidder i with a partition $\mathbf{P}_i$ of the set $\{(j, k): j \in I \text{ and } k \in K\}$ referred to as the analogy partition of bidder i. A typical element of $\mathbf{P}_i$ is denoted by α_i and referred to as an analogy class of bidder i. The element of $\mathbf{P}_i$ that contains (j, k) is denoted by $\alpha_i(j, k)$. The interpretation of $\mathbf{P}_i$ is that bidder i gets informed only of the empirical distribution of bids of bidders engaged in past auctions where all bids b_j submitted in M_k with $(j, k) \in \alpha_i$ are treated alike (i.e., they are not distinguished).

I further assume that a steady state (in which new sets of bidders with newly drawn valuations arrive each time) has been reached so that the empirical distributions of previous bids correspond to the actual distributions. When making his choice of strategy in auction format M_k, bidder i is thus assumed to know only (in addition to $\varphi_i^k(b)$ and $\tau_i^k(b)$) the aggregate distribution of bids in every α_i. He is further assumed to play a best response to the conjecture that bidder j in format M_k bids according to the aggregate distribution of bids in $\alpha_i(j, k)$, the analogy class to which (j, k) belongs.

Formally, let $\mathbf{A} = (M_k, \lambda_k, \mathbf{P}_i)_{i \in I, k \in K}$ denote an auction design. The solution concept is defined as follows.

Definition 1. An *analogy-based expectation equilibrium* of $\mathbf{A} = (M_k, \lambda_k, \mathbf{P}_i)_{i \in I, k \in K}$ is a strategy profile $\beta = (\beta_i)_{i \in I}$ such that for every k and v_i,

$$\beta_i^k(v_i) \in \arg\max_{b_i} u_i^k(v_i, b_i; \overline{\beta}_{-i}^k),$$

[8]Strictly speaking, allowing for mixed strategies $\beta_i^k(v_i)$ should be a distribution over bids. Yet, for my purpose, considering pure strategies is enough.

where $\overline{\beta}^k_{-i} = (\overline{\beta}^k_j)_{j\neq i}$ and $\overline{\beta}^k_j$ is the aggregate distribution of bids in $\alpha_i(j,k)$. That is, $\overline{\beta}^k_j$ is the distribution of bids that assigns weight $\lambda_{k'}/\sum_{(j'',k'')\in\alpha_i(j,k)}\lambda_{k''}$ to the distribution $\beta^{k'}_{j'}(v_{j'})$ as generated by the density $f_{j'}(\cdot)$ for every $(j',k')\in\alpha_i(j,k)$, and the distributions $\overline{\beta}^k_j$, $j\neq i$, are perceived by bidder i to be independent of each other.

Remarks. (i) It should be mentioned that the feedback received by bidders is about the distribution of individual bids and not about the distribution of bid profiles.[9] (ii) The analogy-based expectation equilibrium was first introduced in Jehiel (2005) for extensive form games and in Jehiel and Koessler (2008) for static games of incomplete information. It is further interpreted in the context of private value auctions in Section 2.2, and it is related to other approaches in the literature, in particular, the self-confirming equilibrium in Section 5.1.

Various objectives for the designer are considered. The first objective is a lexicographic criterion with welfare ranked first and revenues ranked second. The second objective is the seller's expected utility as measured by the revenue she gets when there is a transaction and her valuation v_s when there is no transaction. In all cases, the designer is assumed to be risk neutral. She assesses an auction design $\mathbf{A} = (M_k, \lambda_k, \mathbf{P}_i)_{i\in I, k\in K}$ according to the expected value of her objective, assuming that bidders behave according to an analogy-based expectation equilibrium of $\mathbf{A}$.[10]

2.1 *Examples of analogy partitions and auction designs*

The class of analogy partitions considered in Definition 1 is quite large, as it allows for both the bundling of bidders and the bundling of formats in any possible way. For the possibility results that show that the designer can do strictly better than using the finest analogy partition, I either consider the bundling of bidders (Propositions 1 and 2) or the bundling of formats (Proposition 3), but not a combination of the two. The more general formulation allowed by Definition 1 is useful for the impossibility result (Proposition 4), expressing that the full information benchmark is a strict upper bound on what the designer can hope to achieve whatever the manipulation.

Specifically, the following classes of auction designs with public feedback (all $\mathbf{P}_i$ are the same across i) play a central role in the analysis of Propositions 1, 2, and 3.

Class 1: Bidder-anonymous analogy partition. In this case, there is only one auction format, and the feedback is about the aggregate distribution of bids across all bidders. That is, $K = \{1\}$ and for all $i \in I$, $\mathbf{P}_i = \{\bigcup_{j\in I}\{(j,1)\}\}$. For example, the

[9]Accordingly, every bidder i treats every bidder j's distribution of bids, $j\neq i$, as being independent of each other. This fits in well when bidders do not have access (or pay attention to) whether the past bids they observe were submitted at the same or different times.

[10]As in Myerson (1981), one also implicitly assumes that the designer can choose the analogy-based expectation equilibrium she likes best. Yet, which analogy-based expectation equilibrium is played turns out to be inessential for the main results provided weakly dominated strategies are never played.

object could be sold through a first-price auction and bidders would receive feedback about the aggregate distribution of bids with no reference to the characteristics of the bidders who generated the various bids. This is the situation studied in Section 3.

Class 2: Format-anonymous analogy partition. In this case, bidders know the aggregate distribution of bids across the different auction formats M_k, $k \in K$, but they differentiate the distribution of bids for the various bidders $i \in I$. That is, for all $i \in I$, $\mathbf{P}_i = \{\bigcup_{k \in K}\{(j,k)\}\}_{j \in I}$. This situation corresponds to the contest application mentioned in the Introduction and it is considered in Section 4.

2.2 *Interpretation*

The interpretation of an analogy-based expectation equilibrium is that it stands for the limiting outcome of a learning process in which (1) at every stage there is a new auction and new bidders of observable characteristics $i \in I$,[11] (2) auction format M_k is used with frequency λ_k, and (3) a bidder with characteristic i receives as feedback the aggregate empirical distribution of past bids in every α_i.[12] If behaviors stabilize in such a learning process, it must be to an analogy-based expectation equilibrium, provided bidders consider the simplest theory that is consistent with the feedback they receive.

The mechanism design perspective considered in this paper corresponds to the idealization that a single designer can optimize on the auction formats M_k, their frequencies λ_k, and the analogy partitions $\mathbf{P}_i$ provided to bidders, and that behaviors have stabilized to a corresponding analogy-based expectation equilibrium of $\mathbf{A}$. As mentioned in the Introduction, such a view is appropriate in situations in which a single seller repeatedly sells similar objects or in situations in which a single organization repeatedly organizes contests for promotion. It may also be useful to understand the incentives of an auction house such as eBay, which is obviously interested in increasing the sellers' revenues (through the fees they generate), and which can control both the formats and the feedback about past auctions that is disclosed to bidders.

The approach developed in this paper has a non-Bayesian element in the sense that upon learning the coarse feedback the designer reports to them, bidders do not update their belief about the distribution of others' bids based on some (possibly subjective) prior. Instead, bidders are assumed to consider the simplest theory consistent with the feedback they receive: They play a best response to the conjecture that the distribution of bids is uniformly the same over the various (j,k) that are bundled in the same analogy class.[13] I believe this is a natural assumption in many practical situations of interest in which (1) subjects would have no other data than the feedback they receive to form their

[11] The profile of bidders' characteristics is assumed to stay the same throughout the process (see Section 5 for some elaboration on the case in which the number of bidders may vary stochastically).

[12] That is, a bidder with characteristic i is informed of the aggregate empirical distribution of past bids $\{b_j^k, (j,k) \in \alpha_i\}$ with no reference to which (j,k) generated the bid.

[13] Technically speaking, the analogy-based expectation equilibrium can be viewed as a refinement of some variant of the self-confirming equilibrium in which bidders adopt the "simplest" conjecture as just described; see elaborations in Section 5.1.

prior,[14] and (2) the functions φ_j^k and τ_j^k that govern bidder j's incentive in format M_k are either the same across all (j,k) that belong to the same analogy class or they are not known to subjects with characteristic i, $i \neq j$.[15] In such situations, it would seem rather hard (in fact impossible) for subjects to understand how the distribution of bids varies across different (j,k) that belong to the same analogy class simply based on the aggregate empirical distribution they are informed about: assuming that the distribution is the same across these various (j,k) seems focal and I propose it gives a good account for bidders' mode of thinking in such situations.

2.3 *Preliminaries*

A few preliminary observations follow. First, by picking a single auction format M and by adopting the finest analogy partition, the designer can always replicate the revenue generated in M when behaviors are assumed to be governed by Nash equilibrium. Thus, if the designer seeks to maximize revenues, she can always achieve a revenue at least as large as Myerson's (1981) optimal revenue. The question is whether she can achieve larger revenues.

Second, consider an auction format M in which bidder i has a dominant strategy. Then in any auction design including format M, an analogy-based expectation equilibrium requires that bidder i plays his dominant strategy in M (remember that bidders are informed of the allocation rule and the payment rule that applies to them in the format they are in). This is a straightforward observation, since bidder i finds his strategy best no matter what his expectation about the distribution of others' bids is and thus no matter how the auction design is further specified.

Third, one of the auction designs that is studied falls in the following class. There is one auction format M, which respects the anonymity of bidders. That is, consider two bid profiles b and b' obtained by permuting the bids of players i and j. Then $\varphi_i(b) = \varphi_j(b')$ and $\tau_i(b) = \tau_j(b')$, and for all $m \neq i, j$, $\varphi_m(b) = \varphi_m(b')$, $\tau_m(b) = \tau_m(b')$. Consider the *bidder-anonymous analogy partition* defined above, and call $\mathbf{A}$ the corresponding auction design. One can relate the analogy-based expectation equilibria of $\mathbf{A}$ to the Nash Bayes equilibria of the game $\Gamma^{\text{ba}}(\mathbf{A})$ defined by the auction format M in which, for each i, the distribution of bidder i's valuation has the average density $\overline{f}(v_i) = \sum_{i \in j} f_j(v_i)/n$ instead of $f_i(v_i)$.

Claim 1. *A symmetric strategy profile is an analogy-based expectation equilibrium of* $\mathbf{A}$ *if and only if it is a Bayes Nash equilibrium of* $\Gamma^{\text{ba}}(\mathbf{A})$.

Fourth, another class of auction designs $\mathbf{A}$ considered below is such that the various auction formats M_k in $\mathbf{A}$ satisfy $\varphi_i^k(b_i, b_{-i}) = \varphi_i(b_i, b_{-i})$ for all $k \in K$ (for example, in all formats, the object is allocated to the player who submitted the highest bid). When the

[14] This in particular requires that bidders have no prior knowledge about the densities f_i, not even about the frequencies λ_k.

[15] In the first-price auction with bidder-anonymous analogy partition, the former property applies (ϕ_j^k and τ_j^k are the same across all bidders). It may be argued that in the contest for promotion example mentioned in the Introduction, the latter property is often met given that the criteria used for different promotions are often not very transparent to outsiders.

format-anonymous analogy partition prevails, one can relate the analogy-based expectation equilibria of such auction designs $\mathbf{A}$ to the Nash Bayes equilibria of the following game referred to as $\Gamma^{\text{fa}}(\mathbf{A})$.

GAME $\Gamma^{\text{fa}}(\mathbf{A})$. Each bidder i (simultaneously) submits a bid b_i. The object is assigned to bidder i with probability $\varphi_i(b_i, b_{-i})$. Prior to bidding, bidder i is privately informed of his valuation v_i drawn from $f_i(\cdot)$ and of his method of payment k defined by $\tau_i^k(b_i, b_{-i})$. The methods of payment k are identically and independently drawn across bidders and every bidder i is subject to the method of payment k with probability λ_k.[16]

CLAIM 2. *Suppose that the format-anonymous analogy partitions prevail and that in all auction formats M_k of $\mathbf{A}$, $\varphi_i^k(b_i, b_{-i}) = \varphi_i(b_i, b_{-i})$ for all $k \in K$ and $i \in I$. Then a strategy profile β is an analogy-based expectation equilibrium of $\mathbf{A}$ if and only if it is a Bayes Nash equilibrium of $\Gamma^{\text{fa}}(\mathbf{A})$.*

3. Efficiency and revenues

Assume the designer's valuation v_s is 0 and that the designer is interested in both efficiency and revenues, and suppose that the primary objective of the designer is efficiency while revenue is only the secondary objective. In the standard Myerson's paradigm, the so-called revenue equivalence result holds. That is, if two mechanisms result in the same allocation rule and the expected payment made by any bidder i with minimal valuation $v_i = c$ is 0, then both mechanisms must yield the same revenues. Since an efficient outcome can be achieved by a second-price auction SPA, the standard approach (i.e., relying on Nash equilibrium) concludes that the designer can do no better than using a SPA.

I now observe that, within the framework introduced in Section 2, the designer can sometimes achieve strictly larger revenues (than that obtained through the SPA) while still preserving efficiency, thereby illustrating a failure of the allocation equivalence in a manipulative auction design setup. Besides, this gain in revenues is achieved by using a fairly standard auction format (with the bidder-anonymous analogy partition).

PROPOSITION 1. *Consider a two-bidder $i = 1, 2$ auction setup with asymmetric distributions ($F_1(\cdot) \neq F_2(\cdot)$ on a set of strictly positive measure). There is a unique analogy-based expectation equilibrium of the first-price auction with bidder-anonymous analogy partition. Moreover, this analogy-based expectation equilibrium induces an efficient outcome and it generates a strictly higher revenue than the second-price auction. The revenue gain is*

$$\int_c^d \tfrac{1}{4}(F_1(v) - F_2(v))^2\, dv + \int_c^d \tfrac{1}{4}\frac{d\beta(v)}{dv}(F_1(v) - F_2(v))^2\, dv > 0,$$

where $\beta(v) = \int_c^v x\overline{f}(x)\, dx/\overline{F}(v)$, $\overline{f}(x) = (f_1(x) + f_2(x))/2$ and $\overline{F}(v) = \int_c^v \overline{f}(x)\, dx$.[17]

[16]Compared to the true auction design, the difference is that the methods of payment are independently distributed across bidders in Γ^{fa} whereas they are (perfectly) correlated in Γ.

[17]The function $\beta(\cdot)$ is the equilibrium bid function in a symmetric two-bidder first-price auction with density of valuations $\overline{f}$. As such, $\beta(\cdot)$ is an increasing function.

What is the intuition for the above result? First, observe that the use of the bidder-anonymous analogy partition leads the bidders (whatever their characteristic) to best respond to the same distribution of bids, which given the anonymous character of the first-price auction, ensures efficiency. Second, the use of the bidder-anonymous analogy partition leads the bidders to feel that they are in competition with a fictitious bidder who has a distribution of valuations that is the average distribution between the distributions of the various bidders (this essentially follows from Claim 1 above). In the two-bidder case, the price level in the second-price auction is determined by the lowest valuation, hence by the weak bidder. Roughly, the manipulation generated by the bidder-anonymous analogy partition enhances revenues because it makes the strong bidder feel that the weak bidder is less weak than he really is.

When there are more than two bidders, the first-price auction with bidder-anonymous analogy partition remains efficient, but the revenue comparison with the second-price auction can go either way, depending on the form of the asymmetry of the distributions.[18] The following result in which R^{SPA} denotes the expected revenue generated in the second-price auction with densities of valuations $f_i(\cdot)$, $i \in I$, and $\overline{R}$ denotes the expected revenue generated in the fictitious second-price auction with symmetric densities of valuations $\overline{f}_i(v) \equiv \overline{f}(v) = (f_1(v) + f_2(v) + \cdots + f_n(v))/n$ provides a generalization of Proposition 1 to the n-bidder case.

Proposition 2. *Consider an n-bidder setup and assume that $\overline{R} \geq R^{\mathrm{SPA}}$. There is a unique analogy-based expectation equilibrium of the first-price auction with bidder-anonymous analogy partition. Moreover, this analogy-based expectation equilibrium induces an efficient outcome and it generates a strictly higher revenue than the second-price auction.*

Proposition 2 is proven by noting that (1) the revenues in the second-price auction or in the first-price auction with symmetric densities coincide (this is the standard revenue equivalence result) and that (2) the expected revenue in the first-price auction with bidder-anonymous analogy partition is strictly above the expected revenue in the first-price auction with symmetric densities of valuations $\overline{f}(v) = (f_1(v) + f_2(v) + \cdots + f_n(v))/n$ where the latter result follows because (i) bidders employ the same strategy in both cases (by Claim 1) and (ii) the distribution of highest valuation in the asymmetric case first-order stochastically dominates the distribution of highest valuation in the

[18] To see that revenues can go either way, consider first a situation with two bidders whose distribution of valuations is concentrated around d and a third bidder whose distribution of valuation is concentrated around c. It is readily verified that the first-price auction with bidder-anonymous feedback partition generates less revenues than the second-price auction (which achieves a revenue approximately equal to d). Consider next a situation with one bidder whose distribution of valuations is concentrated around d while other bidders have a distribution of valuations concentrated around c. The first-price auction with bidder-anonymous feedback partition generates more revenues than the second-price auction (which generates a revenue very close to c). Thus the revenue comparison can go either way.

symmetric case. While $\overline{R} \geq R^{\mathrm{SPA}}$ always holds in the two-bidder case (thereby explaining why Proposition 1 holds with no restriction on $f_1(\cdot)$ and $f_2(\cdot)$), this need not be so in the more than two-bidder case.[19,20]

The above insights (Propositions 1 and 2) are distinct even though related to Myerson's insight about how to increase revenues in asymmetric auctions. An important implication of Myerson's analysis is that in the asymmetric case, competition between bidders should be biased in favor of weak bidders so as to increase revenues. The net effect of such biased auctions is that some inefficiencies are induced, letting the weak bidder sometimes win the object. As Proposition 1 (and to some extent Proposition 2) illustrates, the use of the bidder-anonymous feedback partitions allows one, in some cases, to symmetrize (a bit) the competition without sacrificing on efficiency. Of course, this is achieved by moving away from the Nash equilibrium paradigm, which the use of partial feedback permits.[21]

Comment. In some applications, the distribution of winning bids as opposed to the aggregate distribution of all bids is available to bidders. From this feedback, bidders can compute a best response strategy based on the assumption that all bidders bid according to the same distribution (this might be argued to be the simplest conjecture in this case). Applied to the asymmetric first-price auction format, the analog of Claim 1 would reveal that in this case bidders would bid as if the distribution of valuations of each bidder were $F(v) = (\prod_{i=1}^{n} F_i(v))^{1/n}$.[22] It is interesting to note that in this case, no matter how many bidders are around, revenues would be higher in the first-price auction (in which only the winning bids are observed) than in the second-price auction.[23]

[19] Intuitively, one would expect to have $\overline{R} \geq R^{\mathrm{SPA}}$ when there is one stronger bidder who is facing symmetrically weak bidders, but the required notion of strong bidder does not boil down to the first-order stochastic dominance relation in the distributions of valuations.

[20] It should also be noted that when the distributions of valuations are nearly the same across bidders (say the cumulative functions differ up to ε), then the revenues of the two auction designs differ according to a smaller magnitude (of order ε^2). When the distributions are very asymmetric, the difference of revenues can be quite substantial. For example, in the two-bidder case considered in Proposition 1, assume that the distribution of valuations of one bidder is concentrated around d, whereas the distribution of valuations of the other bidder is concentrated around c. In this case, the first-price auction with bidder-anonymous feedback partition provides a revenue of $(3c + d)/4$, which should be compared with the revenue c of the second-price auction. Clearly, as d gets large relative to c, the revenue gain, $(d - c)/4$, can be quite substantial in such asymmetric setups.

[21] There is some experimental evidence that in asymmetric first-price auctions, the observed bidding strategy is less asymmetric than Nash equilibrium requires (see Güth et al. 2005). Such an experimental finding is consistent with the view that bidders when considering which bid to submit may look at previous bids without paying attention to the strength of the bidder who submitted the bid (which was modeled here through the apparatus of the bidder-anonymous analogy partition).

[22] Indeed, symmetry implies that all bidders bid according to the same increasing bid function. Thus, only the bid of the highest valuation bidder would be observed, thereby providing the desired result.

[23] The analog of Step 2 in the proof of Proposition 1 is Theorem 1 in Cantillon (2008), and Step 3 holds with an equality because the distribution of the highest valuation is the same with $(F_1, \ldots, F_n)$ and $(F, \ldots, F)$.

4. Optimal auctions

Assume now that the designer seeks to maximize her expected payoff as measured by the revenue she gets when there is transaction and her valuation v_s when there is no transaction. Proposition 3 comprises our first main observation.[24]

Proposition 3. *The largest expected payoff that the designer can achieve in a manipulative auction design is strictly larger than her expected payoff in Myerson's optimal auction (denoted by R^M).*

The intuition for Proposition 3 is as follows. Myerson's optimal auction can always be implemented in such a way that every bidder has a (weakly) dominant strategy and ex post quitting rights of bidders are fulfilled (think of the second-price auction with well chosen reserve price in the symmetric regular case). One can now think of an auction design in which this auction format—call it Myerson design (MD)—is mixed with a little bit of first-price auction (FPA) with v_s reserve price, and bidders get to know only the aggregate distribution of bids over the two auction formats. In format MD, the strategies are the same as in the standard case (because bidders have a weakly dominant strategy in MD). In format FPA, bidders play a best response to the aggregate distribution of bids over the two formats. For many choices of MD, this construction need not induce an expected payoff to the seller that is higher than R^M.[25] But there are many variants of MD in which submitted bids are first monotonically transformed before the original format is applied. For a suitable choice of such a variant, the construction leads bidders in FPA to bid very aggressively because they are led to think that by shading their bid too much, the chance of winning in FPA gets too small. In the limit, a bidder with valuation v may be induced to bid very close to v whenever $v > v_s$. Given that such bidding strategies in FPA induce an expected payoff to the seller that is close to the full information optimal expected payoff $R^F = E(\max_i(v_i, v_s))$ and given that $R^F > R^M$, the result of Proposition 3 follows.

Proposition 3 establishes that the designer can do better than using Myerson's optimal auction (with fine analogy partitions), but how much can she gain? Clearly, the best that she can hope to get in auction designs with ex post quitting rights cannot exceed the maximal full information expected payoff $R^F = E(\max_i(v_i, v_s))$. This trivially follows from the observation that a winner of the auction would always object if he were asked to pay more than his valuation. As it turns out, the designer's best expected payoff in our manipulative design setup lies strictly below R^F.

Proposition 4. *The best expected payoff that the designer can achieve in a manipulative auction design with ex post quitting rights is strictly smaller than the full information expected payoff R^F if $c < v_s < d$.*

[24] By inspecting the proof of Proposition 3, one can see that all analogy-based expectation equilibria (not employing weakly dominated strategies) of the auction design considered there are such that the designer obtains higher revenues than in Myerson's optimal auction. Thus, the conclusion of Proposition 3 would hold under the stronger full implementation requirement (provided one restricts oneself to equilibria not employing weakly dominated strategies).

[25] It can be checked, for example, that in the case of uniform distributions, a mix of second-price auctions and first-price auctions would have no effect on revenues.

In the proof of Proposition 3, some (Myerson-optimal) mechanism MD implementable in (weakly) dominant strategy was mixed with a little bit of first-price auction with reserve price v_s, FPA, and the seller's expected payoff obtained in FPA was shown to be close to R^F. However, such a construction requires that the frequency with which FPA is used is set sufficiently small. As one increases the frequency of FPA, the manipulation loses its force and, of course, in the limit as the designer almost always picks FPA, one gets the standard seller's expected payoff generated in the first-price auction with reserve price v_s, which following Myerson's analysis cannot be greater than R^M.

What Proposition 4 establishes is that within the class of mechanisms under study one can never reach R^F whatever the manipulation. This observation is of particular interest because if the designer could freely choose the belief of bidders (with no constraint), she could get a payoff arbitrarily close to R^F (see the arguments surrounding Proposition 7). Thus, Proposition 4 establishes that there is some limitation to manipulation imposed by the requirement that the feedback should be correct (even if coarse).

To get an intuition for Proposition 4, think of a symmetric scenario in which the auction design **A** uses the format-anonymous feedback partition. To get close to R^F, it would be required that, in all auction formats M_k used in **A**, every bidder with valuation $v > v_s$ pays a price close to his valuation when he wins. This implies that every bidder should perceive making almost zero profit.[26] Yet a bidder with valuation $v \simeq d$ can always consider submitting a bid $\beta^k((d+v_s)/2)$ in format M_k, and he should expect there to win and make a payment no more than $(d+v_s)/2$, thereby making a net gain of no less than $(d-v_s)/2 > 0$ whenever other bids b_j are smaller than $\beta^k((d+v_s)/2)$.[27]

Now, it is not a priori clear what perceived probability bidder i attaches in format M_k to the event that all b_j, $j \neq i$, are smaller than $\beta^k((d+v_s)/2)$. But one can always rank the formats M_k by increasing order of $\beta^k((d+v_s)/2)$. For those formats M_{k^*} such that $\beta^{k^*}((d+v_s)/2)$ is above the median of $\beta^k((d+v_s)/2)$ (in the distribution in which $\beta^k((d+v_s)/2)$ has probability λ_k), it is clear that in the format-anonymous analogy partition, this perceived probability is no less than $\frac{1}{2}\Pr(\max_{j\neq i} v_j < (d+v_s)/2)$.

A contradiction is obtained given that, in format M_{k^*}, a bidder with valuation $v \simeq d$ cannot perceive his payoff from following $\beta^{k^*}(d)$ (this should be approximately 0) to be strictly smaller than his payoff from following $\beta^{k^*}((d+v_s)/2)$ (this has been shown to be no less than $\frac{1}{2}(d-v_s)/2\Pr(\max_{j\neq i} v_j < (d+v_s)/2)$, which is strictly positive).

Comments. (1) From the proof of Proposition 4, if $v_s \leq c$, it is not clear whether a similar conclusion arises, because it is not then a priori guaranteed that for the full information payoff to arise, a bidder should necessarily perceive making negligible profit. It should be noted though that with the additional requirement that in every format M_k, the payment made by the winner should lie in between the largest and the second largest bid,

[26]The perceived payoff might a priori differ from the actual one due to the manipulative character of the design. To establish this, I make use of $v_s > c$ and of the monotonicity of $\tau^k(b_i, b_{-i})$, but I suspect this holds much more generally.

[27]This is because in format M_k, the actual bid of bidder i with valuation $v < (d+v_s)/2$ is $\beta^k(v)$, and a bidder with valuation $(d+v_s)/2$ should win and pay a price approximately equal to his valuation whenever he meets bidders with lower valuations.

then the same conclusion as in Proposition 4 would arise even if $v_s < c$, because the rules of the formats would imply that it is weakly dominated to bid above one's own valuation (and this property can easily be used to establish that bidders cannot perceive making significant gains if R^F is to be achieved). (2) If the designer were allowed to commit to offering positive payments to losers and if the payments from the winner were not assumed to be monotonic in bids, then the designer could get a revenue close to R^F while still preserving the ex post participation constraints of bidders.[28] The restriction on mechanisms (i.e., not allowing positive payments to losers and imposing that payments from winners be monotonic in bids) can then be thought of as resulting from the regulatory desire to protect bidders from manipulation.

5. Discussion

5.1 *Related literature*

This paper is related to several strands of literature. First, the concept used in this paper follows Jehiel (2005), Jehiel and Koessler (2008), and Ettinger and Jehiel (2010) who define the analogy-based expectation equilibrium for extensive form games of complete information, static games of incomplete information, and multistage games of incomplete information, respectively. The analogy partitions considered in this paper are slightly less general than those considered in Jehiel and Koessler (2008) except for the fact that bidder *i*'s own bids can be lumped together with others' bids in bidder *i*'s analogy partition.[29] The main novelty of the approach pursued here is that the feedback partitions are viewed as a choice made by the designer. That is, they are not exogenously given as in Jehiel (2005) or Jehiel and Koessler (2008).[30]

[28] To see this, consider a symmetric two-bidder scenario in which bidders' valuations are identically distributed on (c, d). Consider an auction design with format-anonymous analogy partition and two formats $\underline{M}$ and $\overline{M}$ used in equal proportion. In format $\underline{M}$, the equilibrium bids lie in $[0, d]$; in format $\overline{M}$, the equilibrium bids lie in $\{0\} \cup [d, 2d]$. In each format, a bidder with negative valuation bids 0 in equilibrium. In both $\underline{M}$ and $\overline{M}$, the bidder with highest bid wins the auction if this bid is strictly positive. In $\underline{M}$, if $b_i \in (0, d)$ for $i = 1, 2$, the winner pays his own bid and the loser receives no transfer. In $\overline{M}$, if $b_i \in \{0\} \cup [d, 2d]$ for $i = 1, 2$, the winner i^* pays $b_{i^*} - d$ and the loser receives no transfer. The idea is to augment the transfers in $\underline{M}$ and $\overline{M}$ to cover all bid profile configurations, even for bid realizations that never occur in the respective formats. So in $\underline{M}$, a (losing) bidder submitting $b_i \in (0, d)$ is offered a promise of transfer $\underline{h}(b_i)$ if $b_j \in (d, 2d)$, and in $\overline{M}$, a (winning) bidder submitting $b_i \in (d, 2d)$ is offered a transfer $\overline{h}(b_i)$ if $b_j \in (0, d)$. By suitable choices of $\underline{h}$ and $\overline{h}$, one can ensure that for $v_i > 0$, bidding $\underline{\beta}(v_i) = v_i$ in $\underline{M}$ and bidding $\overline{\beta}(v_i) = v_i + d$ in $\overline{M}$ is an analogy-based expectation equilibrium. [For example, in the uniform distribution case, $\underline{h}(b) = b^2/(2d) - bc/d$ and $\overline{h}(b) = (b - d)^2/(2d) - (b - d)c/d$. These functions are determined so that the expected perceived transfers correspond to those that would be made in a SPA with 0 reserve price.] With such bidding strategies, the expected revenues generated in each format are R^F, and thus the designer gets R^F in expectation.

[29] As already mentioned, such an extension is particularly adapted if each individual bidder *i* participates in just one auction.

[30] By assuming that the designer can control the analogy classes of the bidders, this paper takes the more rational interpretation of the concept, that is, assuming that players make maximal use of the information provided to them. In Jehiel (2005), it is alternatively suggested that an analogy-based expectation equilibrium might arise because players may not pay attention to all of the details of their past experiences, which amounts to a bounded rationality interpretation of the concept.

The analogy-based expectation equilibrium has a close relationship to some variant of the self-confirming equilibrium in which bidder i would receive as signal the aggregate distribution of bids in α_i for the various $\alpha_i \in P_i$ (see Fudenberg and Levine 1998 for a general presentation of the self-confirming equilibrium).

Specifically, suppose at the end of an auction round described by the format M_k, and the profile of bids and the profile of valuations (b, v, k), subjects of population i receive as feedback various signals $y^z_{s_i}(b, v, k) \in [0, d]$ for $z = 1, \dots, n(s_i, k)$ and $s_i \in S_i$, where $n(s_i, k)$ denotes the number of signals of type s_i received when the format is M_k. If bidders follow the strategy β and formats are drawn according to λ (see Section 2), then

$$\Pr_{\beta,\lambda}(y_{s_i} \in X) = \left(\sum_k \lambda_k \sum_{z=1}^{n(s_i,k)} \Pr_{\beta}(y^r_{s_i} \in X)\right) \Big/ \left(\sum_k \lambda_k n(s_i, k)\right)$$

is the empirical frequency with which a signal y_{s_i} falls in X for every $X \subseteq [0, d]$. A self-confirming equilibrium allows bidders i to entertain subjective views about λ and β, but it requires that bidders play best responses given these subjective views and that the empirical frequencies as computed from the subjective views coincide with the correct empirical frequencies of y_{s_i} as generated by λ and β. The formal statement follows.

Definition 2. A strategy profile β is a *self-confirming equilibrium given y* if for every bidder i, there exist conjectures $\widetilde{b}^i = (\widetilde{b}^{i,k}_i, \widetilde{b}^{i,k}_{-i})_k$ and $\widetilde{\lambda}^i = (\widetilde{\lambda}^i_k)_k$ such that for every k and every $v_i \in (c, d)$,

$$\beta^k_i(v_i) = \arg\max_{b_i} u^k_i(v_i, b_i; \widetilde{b}^{i,k}_{-i}),$$

and for every $s_i \in S_i$ and $X \subseteq [0, d]$,

$$\Pr_{\beta,\lambda}(y_{s_i} \in X) = \left(\sum_k \widetilde{\lambda}^i_k \sum_{z=1}^{n(s_i,k)} \Pr_{\widetilde{b}^i}(y^r_{s_i} \in X)\right) \Big/ \left(\sum_k \widetilde{\lambda}^i_k n(s_i, k)\right). \tag{1}$$

Compared to the usual definition of self-confirming equilibrium, it is somehow nonstandard that several realizations of signals of type s_i would be observed in a given auction round. It is also nonstandard to require the consistency of the conjectures only with respect to the marginal distributions of y_{s_i} (as expressed in (1)) as opposed to the joint distribution over all signals received by player i. These differences are in part motivated by the fact that I have in mind situations in which an individual player plays only once, whereas the literature that introduced the self-confirming equilibrium focused on situations in which players would play many times.[31]

Equipped with this notion of self-confirming equilibrium, by identifying s_i with an analogy class α_i and identifying the various $y^r_{s_i}(b, v, k, t)$ with the various b_j in format M_k such that $(j, k) \in \alpha_i$, it is readily verified that an analogy-based expectation equilibrium (Definition 1) is a self-confirming equilibrium given this signal structure.

[31] Requiring only the consistency of the marginal distributions corresponds to the idea that players do not keep track of when previous realizations of signals occurred.

As was anticipated in Section 2.2, an analogy-based expectation equilibrium is a selection of self-confirming equilibrium,[32] the one in which the theory or conjecture adopted by bidders is the simplest in the sense that their theory has the coarsest measurability property while being consistent with the feedback.[33] More precisely, the analogy-based expectation equilibrium under the stipulation that player i best responds to the conjecture that for every $(j,k) \in \alpha_i$, the distribution of bids of bidder j in format M_k coincides with the aggregate distribution of bids in α_i, is such that player i's theory about the play of bidder j in M_k is the same across all $(j,k) \in \alpha_i$. That is, identifying the state space of theories with $\{(j,k), j \in I, k \in K\} \times \Delta_r$, where Δ_r is the r-dimensional simplex (representing probability distributions over the various formats M_k, $k = 1, \dots, r$), player i's theory in an analogy-based expectation equilibrium is measurable with respect to his analogy partition, and any theory that is consistent with the feedback would have to be at least as fine.[34]

The literature on self-confirming equilibrium has not considered feedback from the players as a design issue, with the notable exception of Esponda (2008a). Esponda considers first-price auctions in which the *same* bidders get involved over sequences of auctions, and get information about the joint distribution of highest bids (and possibly second-highest bids) and their own valuation and bid. In a symmetric first-price auction with private and affiliated values he shows that *symmetric* self-confirming equilibria (of the static auction) generate at least as much revenue as the Nash equilibrium.

The main common aspect of the two papers is that the solution concept that we use is not Nash equilibrium, but some concept related to the self-confirming equilibrium, and that it is applied to a mechanism design question. Yet, there are several important differences between Esponda (2008a) and this paper that I now discuss.

First, by allowing for a large class of auction formats, I am able to place my analysis in the context of the optimal auction design literature (Myerson 1981), which would not have been possible if I were to confine myself to first-price auctions. Second, motivated by some applications such as eBay, I have considered here a case in which fresh bidders arrive each time, which requires amending the usual definition of self-confirming equilibrium (see Definition 2). More importantly, I consider a refinement of self-confirming equilibrium based on simplicity considerations. In general, when one looks at all self-confirming equilibria, it is hard to develop clear-cut comparative statics regarding the effect of the feedback (in particular, because the Nash equilibrium is always a self-confirming equilibrium whatever the feedback). For example, in the symmetric context studied by Esponda (2008a), to obtain clear-cut comparative statics, he had to restrict

[32]Dekel et al. (2004) show that in private value settings such as the one in this paper, a self-confirming equilibrium coincides with a Nash equilibrium whenever players can observe the actions of the players. An analogy-based expectation equilibrium in my setup may differ from Nash equilibrium because bidders do not observe other bidders' actions separately in each format M_k in which they may participate.

[33]While such a selection should be the subject of empirical test, the experiment in Huck et al. (2011), gives some support to this selection, even though not in an auction setup.

[34]Observe that this implies that no theory about λ need be specified in an analogy-based expectation equilibrium, which is not so in most other self-confirming equilibria (as soon as the conjecture of the distribution of bids is not the same for (j,k) and (j',k') in the same analogy class, some theory about $\lambda_k/\lambda_{k'}$ is required to check the consistency condition).

himself to *symmetric* self-confirming equilibria, as it is not true that his result would hold if asymmetric self-confirming equilibria were considered. But, in general, one has to provide some motivation for the chosen restriction.[35] In the research started in Jehiel (2005) and continued in this paper, I propose looking at the selection in which players adopt the simplest theory consistent with the feedback they receive (in a sense made precise above), and one might argue that the simplicity criterion is a common theme of the entire literature on bounded rationality (see, for example, Simon 1955), thereby suggesting some motivation for the chosen selection. Equipped with such a selection, I was able to obtain clear-cut comparisons because essentially the chosen selection allowed me to have a comparable predictive power whatever the feedback (which is not so if one works with the entire set of self-confirming equilibria).

Some other equilibrium approaches that move away from Nash equilibrium (and thus permit erroneous expectations) have been proposed in the recent past. These include the cursed equilibrium of Eyster and Rabin (2005) and the behavioral equilibrium of Esponda (2008b). These approaches shed new light on the winner's curse and on the possibility of trade in adverse selection problems. Yet, in private values setting such as the one considered here, these approaches coincide with Nash equilibrium and as such are not closely related to the present study (see Jehiel and Koessler 2008 for a discussion of the link between the cursed equilibrium and the analogy-based expectation equilibrium in general Bayesian games).

There is also a strand of literature concerned with learning in mechanism design. This strand includes among others Cabrales (1999) and Cabrales and Serrano (2007) (see the latter for a more comprehensive review of that strand of literature). A typical question addressed in this literature concerns equilibrium selection when there are several Nash equilibria and whether the choice of mechanism may induce good convergence properties of the corresponding learning process. The approach pursued here is complementary to this strand. It offers a different perspective by suggesting how the use of coarse feedback may result in convergence to non-Nash equilibria, i.e., analogy-based expectation equilibria. The approach pursued here also starts with the assumption that a steady state has been reached. In line with the literature just mentioned, it would be of interest to study the convergence properties of the learning process that was suggested to motivate the present approach.

Finally, there have been various approaches to study how a mechanism designer should deal with various behavioral biases assumed on agents. These approaches include, among others, the work of Eliaz (2002), who assumes that a fixed number of agents may have a crazy behavior, the work of Eliaz and Spiegler (2007), who assume that agents may have erroneous subjective beliefs, and the work of Crawford et al. (2009), who assume that bidders behave according to the level-k mode of thinking.[36] Beyond the obvious observation that the bias considered in this paper is of a different nature

[35] Given that there are several possible symmetric conjectures in Esponda's setup, it is not clear by which process the bidders should be coordinated on a symmetric self-confirming equilibrium, thereby making the restriction to symmetric self-confirming equilibrium questionable.

[36] In a related vein, Matsushima (2008) considers an implementation problem when agents rely on two levels of eliminations of dominated strategies.

than those considered by these authors, I believe the current approach differs from these other approaches in an important way. Somehow the bias in the expectation formation that appears in an analogy-based expectation equilibrium is induced by the choice of analogy partition made by the designer. It is thus as if the cognitive limitations of the bidders were endogenously created by the designer rather than being there to start with.

5.2 *Complete information*

In the above analysis, some uncertainty about bidders' valuations was assumed. When each bidder i's valuation can take a single value v_i, the designer can extract a revenue equal to $R^F = \max_i(v_i, v_s)$ in the classic rationality setup.

In the above setup with ex post quitting rights, no manipulation can allow the designer to extract more than R^F given that a bidder never accepts paying more than his valuation if he wins the object (and he never accepts paying anything if he does not win the object). Thus, within the setup introduced in Section 2, some private information is required for manipulation to be of effective use to the designer.

5.3 *Interim participation constraints*

In the above analysis, auction formats with ex post quitting rights were considered. If participation constraints are required only at the interim stage before bidders know the outcome of the auction and if relatedly the designer can also require payments from losers, then the designer can generate much larger revenues if bidders play according to an analogy-based expectation equilibrium.[37]

Proposition 5. *Suppose there are at least two bidders and that bidders cannot withdraw from the auction later on. Then by a suitable choice of auction design, the designer can make arbitrarily large amounts of money.*

The idea of the proof, which is detailed in the Appendix, is as follows. By choosing several formats and by using a format-anonymous analogy partition for say bidder 1, the designer can mislead bidder 1 in his understanding of the distribution of bids of other bidders $i \neq 1$. She can then propose a bet to bidder 1, whose monetary outcome is contingent on the realization of b_i, $i \neq 1$, in such a way that the bet sounds profitable from the viewpoints of both bidder 1 and the designer. By increasing the stakes of the bet, bidder 1 still agrees on the terms of the bet given our assumption of risk neutrality, which translates into potentially arbitrarily large revenues for the designer.

The above argument bears a strong resemblance to the observation that with subjective prior beliefs, the logic of the no trade theorem breaks down.[38] Of course, here since the designer is assumed to know the correct distributions of bids, one makes the further

[37] An analogy-based expectation equilibrium can be defined in a similar way as in Definition 1 for such auction designs without ex post quitting rights.

[38] See, however, Morris (1994) for an exploration of when the no trade theorem continues to hold in the subjective prior paradigm.

inference that it is the designer (and not the bidder) who benefits from the bet. Another key difference with the literature on subjective priors is that the erroneous perception of the bidders is viewed here as the result of the feedback manipulation of the designer and not the subjective character of bidders' prior beliefs.

Comment. To the extent that bidders know that the designer is more informed than they are about the distributions of bids, one might argue that in the context of the above manipulation, bidder 1 might be suspicious, thereby deciding to stay outside the auction room rather than to play according to an analogy-based expectation equilibrium.[39] This is to be contrasted with auction designs with ex post quitting rights as considered in the main part of this paper in which staying outside the auction room is always a bad idea (nothing worse can happen by participating). In the class of auction designs with ex post quitting rights, it is not clear what else (i.e., other than playing according to an analogy-based expectation equilibrium) a player could do.[40]

5.4 *Further restrictions on the set of mechanisms*

Restrictions on mechanisms beyond those made in Section 2 can be considered. A question of interest is whether with such extra restrictions, the results of Propositions 3 and 4 still hold. For example, consider the case in which, in addition to the assumptions made in Section 2, the payment $\tau_i^k(b)$ made by the winner is required to be in the convex hull generated by the submitted bids (one might argue that such a feature is satisfied by all auctions in the real world so that the use of other auctions might trigger the suspicion of bidders). Then clearly, the result of Proposition 4 would a fortiori hold, since I am restricting the domain on which the designer can choose her auction design. Whether the result of Proposition 3 still holds should be the subject of a more systematic investigation. I provide a special case in which it would hold and I suspect it might hold much more generally.

Proposition 6. *Suppose there are two bidders whose valuations are uniformly distributed on $[c, c+1]$ with $c > v_s + 1$. The seller can do strictly better than in Myerson's optimal auction in a manipulative auction design with ex post quitting rights in which the price paid by the winner is constrained to be in between the bids submitted by the two bidders.*

[39] Alternatively, if one has in mind that there is a risk that bidder 1 would play according to an analogy-based expectation equilibrium even for large stakes, one may think of the ex post quitting rights scenario considered in the main part of the paper as a regulatory constraint imposed on designers to better protect bidders from manipulation.

[40] Recently, Lehrer (2008) proposed a selection of self-confirming equilibrium based on the most pessimistic conjecture (rather than the simplest conjecture as in this paper). Such an approach would lead bidders not to accept bets as considered in the proof of Proposition 5. But it would also lead bidders not to take part in any auction of the sort analyzed throughout this paper as long as feedback is partial and there is a slight cost to participate in auctions. I find the latter conclusion unrealistic.

5.5 *Shill bidding*

In the above analysis, the only players in the auction were the bidders $i \in I$. It might be argued that the designer could also employ shill bidders in addition to the real bidders $i \in I$. In the standard paradigm (i.e., relying on Nash equilibrium), this does not help the designer obtain a better outcome (as results from Myerson's analysis), but in a manipulative mechanism design setup it does, as I now illustrate.

Assume that the designer can costlessly hire shill bidders who have no intrinsic value for the object.[41] Specifically, consider the same setup as in Section 2 except that the designer can also add to the set $I = \{1, \dots, n\}$ of actual bidders any set $S = \{n+1, \dots, n+m\}$ of shill bidders (with 0 valuations). When the seller sells to a shill bidder, I assume her payoff is v_s (this is equivalent to the seller keeping the object and not making any payment). Otherwise, the seller's payoff is as in Section 4. That is, her payoff is the revenue if she sells to a bidder $i \in I$ and is v_s if she does not sell.

The following proposition shows that in a manipulative auction design with ex post quitting rights, the seller can get a payoff close to the full information payoff R^F whenever she can freely hire shill bidders, which should be contrasted with the finding of Proposition 4.

Proposition 7. *Suppose the cost of hiring shill bidders is zero. Then the designer can get a revenue close to R^F in the optimal manipulative auction design.*

The proof of Proposition 7 follows the logic used to prove Proposition 3. By inviting m shill bidders, the designer can make them bid as she wishes, say each according to a distribution of bids $g(\cdot)$ with support on (v_s, d). Consider now a variant of the first-price auction with reserve price v_s defined as follows. Only a bidder $i^* \in I$ can win the auction and he wins if $b_{i^*} = \max_{i \in I} b_i$ and $b_{i^*} \geq v_s$. Otherwise, the seller keeps the object. The shill bidders never win the auction (and thus never make any payment), but they are requested by the seller to bid according to a distribution of bids with density $g(\cdot)$.[42]

Consider now the bidder-anonymous analogy partition in the above auction format in which every bidder gets to know only the aggregate distribution of bids that mixes the bids of bidders $i \in I$ and the bids of shill bidders. It is readily verified that, in an analogy-based expectation equilibrium, as m grows to infinity, bidders $i \in I$ submit a bid that is approximately a best response to the conjecture that bidder $i \in I = \{1, \dots, n\}$ bids according to $g(\cdot)$ (because in the aggregate distribution of bids that mixes the bids of all bidders, the distribution of bids of shill bidders overcrowds the distribution of bids of bidders $i \in I$). That is, each bidder $i \in I$ with valuation $v > 0$ submits a bid close to $\beta(v) \in \arg\max_{b \geq v_s} (v - b) G^n(b)$, where $G(\cdot)$ is the cumulative of $g(\cdot)$. By considering a cumulative $G(\cdot)$ of the form $G(v) = ((v - v_s)/(d - v_s))^q$ with q large enough, one easily obtains that $\beta(v)$ gets close to v, thereby providing a proof of Proposition 7 (see more details in the proof of Proposition 3).

[41] One may argue that in practice the cost of hiring shill bidders is high to the extent that shill bidding is illegal. The following proposition helps understand why it may be a good idea to make shill bidding illegal.

[42] Shill bidders are indifferent as to how they bid given the auction rule, but I assume that they follow the request of the designer. They could easily (and cheaply) be incentivized to do so.

5.6 *Random number of bidders*

In the above analysis, the set of bidder I was deterministic (this was also to simplify the comparison with Myerson's optimal auction paper). How are the results affected if the set of bidders I is stochastic?

For concreteness, consider a symmetric regular case in which the valuation of every bidder i is drawn from the same distribution with density $f(\cdot)$ and $v \to v - (1 - F(v))/f(v)$ is increasing. When bidders are risk neutral as assumed in this paper, the best revenue in the classic rationality setup is achieved by having a regular auction (say a second-price or first-price) auction with reserve price R set such that $R - (1 - F(R))/f(R) = v_s$. This is so because such a format would achieve the best revenue even if the number of bidders were known to the designer and no matter what this number is (see McAfee and McMillan 1987 for the treatment of risk aversion when the number of bidders is stochastic).

Can the designer achieve larger revenues in a manipulative auction design setup when the number of bidders is stochastic? The answer is yes and this is shown in the same way as Proposition 3 was proven, that is, by mixing a little bit of first-price auction with reserve price v_s with a well chosen auction format that is strategically equivalent to the second-price auction with reserve price R, and by considering the format-anonymous analogy partition. Similarly, Proposition 4 extends to the stochastic number of bidder case.

In a vein similar to that of Propositions 1 and 2, one might also be interested in scenarios in which the number of bidders would vary from one auction to the next and bidders would observe the number of competitors they face in the current auction. If a second-price auction is considered, observing the number of competitors has no effect on the optimal strategy, but if a first-price auction is considered, such an observation affects the optimal strategy (as it is indicative of how the chance of winning depends on the bid shading). In the latter case, one may also consider the effect of providing as feedback the distribution of past bids without telling how many bidders were present in the auction room when the bid was submitted. How does the revenue generated in such a first-price auction design compare to the revenue of the second-price auction? Addressing such a question is left for future research.

5.7 *Cheating on feedback*

An important assumption made throughout the paper is that the feedback reported by the designer must be correct. There are important reasons why I believe this is a natural assumption. First, in most countries it is illegal to report false pieces of information (this should be contrasted with the kind of manipulation considered in this paper in which every bit of released information is correct even if partial). Second, even if there is no legislation about the correctness of feedback, it is likely to be in the interest of sellers to report truthful feedback, as otherwise if bidders realize feedback may be erroneous, there is no reason why bidders would trust the feedback that is transmitted to them. Along this line, it may be argued that it is in the interest of an auction house such as eBay to be committed to never report false feedback to bidders (and it seems clear that no one would dispute the correctness of the feedback provided by eBay to bidders).

5.8 *Other forms of feedback*

Even if the class of feedback considered in this paper is quite large, some forms of feedback are not covered. A key difficulty should be addressed though if one wishes to consider more general classes of feedback. That is, one needs to define an appropriate/focal notion of best response to the feedback received by the bidder, thereby leading to an appropriate notion of equilibrium. While this may be defined in some cases beyond the class of feedback considered here (see the discussion at the end of Section 3), I believe this would be often problematic for general forms of feedback as the feedback would not easily translate into a focal conjecture about other players' strategies. In addition, as already mentioned, considering the whole set of self-confirming equilibria is unlikely to give sharp predictions in general.

5.9 *The replacement assumption*

To motivate the analogy-based expectation equilibrium, I have assumed that new bidders participate in each auction (see Section 2.2). It would clearly be of interest to cover also situations in which each individual bidder remains active longer. When bidders remain active arbitrarily long, convergence to Nash equilibrium should be expected in the private values setup considered here given that in each format M_k bidders could learn the distribution of other bidders' bids from their own past observations.[43] However, if each individual bidder does not remain active forever, then some outcome in between the Nash equilibrium and the analogy-based expectation equilibrium (as defined in this paper) should be expected. More work is required to model such intermediate cases in a satisfactory way.

6. Conclusion

I believe the above abstract setup is useful to understand a number of applications. The idea that bidders form their beliefs about others' bidding strategies by looking at the history of past bids should sound familiar to anyone who has considered buying or selling on eBay (under the item "completed listings" one has access to the history of previous bids in auctions of similar objects that took place within a month).[44] The feedback provided on eBay is partial in the sense that one never has access to the characteristics (such as gender, age, etc.) of the bidders and the same feedback appears whether or not a buyout option prevailed, as long as the option was not exerted (technically speaking, whether or not there is a buyout option should be interpreted as corresponding to different auction formats).

What this paper has emphasized is the use of feedback policy as a new instrument in mechanism design. My main interest was to understand the effect of the feedback policy on efficiency and revenues.

[43]This is implicitly assuming that players make the best use of their observations. If, however, their attention is limited, they may still be playing an analogy-based expectation equilibrium in this case.

[44]See Reynolds and Wooders (2009) for an analysis of auctions with buyout options. I am grateful to John Wooders for introducing me to the various formats and feedback available on eBay.

The main contribution of this paper is to show that there is a role for a strategic use of feedback disclosure in mechanism design. On the one hand, first-price auctions with bidder-anonymous analogy partition generate more revenues than second-price auctions when there are two bidders, or more generally, when the revenue of the second-price auction with symmetrized distributions exceeds the revenue in the second-price auction (with asymmetric distributions). Thus, in such cases, providing coarse feedback in first-price auctions may be thought of as a new way to promote more competition in asymmetric auctions that avoids the cost of reducing efficiency. On the other hand, the insight that with coarse feedback one can generate more revenues than in Myerson's optimal auction is suggestive that the lack of transparency that is often observed in promotion-like contests may be desirable for organizations. More work is required to understand more generally how much can be gained with the use of coarse feedback in mechanism design and how one should regulate such markets to protect consumers.[45]

Appendix

Proof of Claim 1. Consider a symmetric analogy-based expectation equilibrium β of **A** (where $\beta(v)$ refers to the equilibrium bid of any bidder with valuation v).[46] By definition, bidder i plays a best response to the distribution of bids of other bidders that assigns density $\sum_{j\in I} f_j(v)/n$ to the bid $\beta(v)$. But this is the definition of a Bayes Nash equilibrium of $\Gamma^{\mathrm{ba}}(\mathbf{A})$. The converse is also immediate. □

Proof of Claim 2. Consider an equilibrium β of $\Gamma^{\mathrm{fa}}(\mathbf{A})$. In $\Gamma^{\mathrm{fa}}(\mathbf{A})$, bidder i whatever his payment method expects every other bidder $j \in I$ to be facing the payment method k' with probability $\lambda_{k'}$, hence to be playing according to strategy $\beta_j^{k'}(\cdot)$ with probability $\lambda_{k'}$. Thus, in $\Gamma^{\mathrm{fa}}(\mathbf{A})$, when the payment method is k, bidder i plays a best response $\beta_i^k(v_i) \in \arg\max_{b_i} u_i^k(v_i, b_i; \overline{\beta}_{-i}^k)$, where $\overline{\beta}_j^k = \sum_{k'} \lambda_{k'} \beta_j^{k'}$ and $\beta_j^{k'}$ is the distribution of bids of bidder j when j has the method of payment k'. But this corresponds exactly to the definition of an analogy-based expectation equilibrium of **A**. The converse is also immediate. □

Proof of Proposition 1.

Step 1. *Consider the first-price auction with bidder-anonymous analogy partition. There exists a unique analogy-based expectation equilibrium defined as follows: for* $i = 1, 2$, $\beta_i(v) = \beta(v) = (\int_c^v x\overline{f}(x)\,dx)/\overline{F}(v)$, *where* $\overline{f}(x) = (f_1(x) + f_2(x))/2$ *and* $\overline{F}(v) = (F_1(v) + F_2(v))/2$. *Bidders never quit ex post and the outcome is always efficient, i.e., the bidder who values the good most gets the object.*

[45] For example, there is an active debate as to whether the Swoopo auction site (which currently includes bids in different currencies as well as bidding fees) should be regulated. In line with the insights developed in this paper, it might make sense to impose the use of a common currency (as otherwise there is a risk that bids made in different currencies may be confused when looking at past auctions).

[46] The anonymity properties of M_1 ensure the symmetry (across bidders) of the best response correspondence.

Proof. Consider an analogy-based expectation equilibrium $\beta_i(\cdot)$ for $i = 1, 2$. Standard incentive compatibility considerations imply that $\beta_i(\cdot)$ must be a nondecreasing function of the valuation (as otherwise a higher valuation type of bidder i would perceive winning the object with a probability strictly lower than a lower valuation type, which is ruled out by incentive compatibility). Thus, the bid functions $\beta_i(\cdot)$ must be continuous almost everywhere.

Suppose we have a nonsymmetric equilibrium (that is not equivalent almost everywhere to a symmetric equilibrium). This implies that for a positive measure of v, $\beta_1(v) \neq \beta_2(v)$ and both $\beta_1(v)$ and $\beta_2(v)$ are best responses for a bidder with valuation v to the aggregate distribution of bids. There must then be a neighborhood of v within which a positive measure of v has this property. Yet, this implies that we can make another selection of the best response correspondence that violates the monotonicity of $\beta_i(\cdot)$, thereby showing a contradiction.[47]

The rest of the argument follows from Claim 1 (see Section 4). Indeed, any symmetric analogy-based expectation equilibrium must be a Nash Bayes equilibrium of the FPA with symmetric bidders and density $\overline{f}(v)$, and vice versa. Given the analysis of the FPA with symmetric bidders, we may conclude as desired. (The fact that bidders never exert their ex post quitting rights follows from the rules of FPA. No bidder finds it optimal to bid above his valuation and thus when he wins, a bidder finds it optimal to accept the deal.) ◁

Call R the revenue generated in the first-price auction with bidder-anonymous analogy partition. Call R^{SPA} the revenue generated in the second-price auction. Finally, call $\overline{R}$ the expected revenue generated in the second-price auction with symmetric bidders and density of valuations $\overline{f}(v) = (f_1(v) + f_2(v))/2$. These revenues are written as (the identity between the last two expressions can be obtained as a consequence of the allocation equivalence theorem)

$$R = \int_c^d \beta(v)[f_1(v)F_2(v) + f_2(v)F_1(v)]\,dv$$

$$R^{\text{SPA}} = \int_c^d vf_1(v)[1 - F_2(v)]\,dv + \int_c^d vf_2(v)[1 - F_1(v)]\,dv$$

$$\overline{R} = 2\int_c^d v\overline{f}(v)[1 - \overline{F}(v)]\,dv$$

$$\overline{R} = 2\int_c^d \beta(v)\overline{f}(v)\overline{F}(v)\,dv.$$

Step 2. $\overline{R} - R^{\text{SPA}} = \int_c^d \frac{1}{4}(F_1(v) - F_2(v))^2\,dv$.

[47] Suppose $\beta_1(v) < \beta_2(v)$. By continuity, $\beta_1(v+\varepsilon) < \beta_2(v)$ and $\beta_2(v+\varepsilon) > \beta_1(v)$. The definition of an analogy-based expectation equilibrium implies that $b_1(v) = \beta_2(v)$ and $b_1(v+\varepsilon) = \beta_1(v+\varepsilon)$ with all other bids unchanged should also be part of an equilibrium. But such bids would violate the incentive compatibility conditions and as a result cannot maximize (over bids) the corresponding expected payoffs of bidder 1 with valuations v and $v+\varepsilon$.

PROOF. Using the first expression of $\overline{R}$, we have that $\overline{R} - R^{\text{SPA}}$ can be written as

$$\int_c^d v\Big[-\tfrac{1}{2}(F_1(v)+F_2(v))(f_1(v)+f_2(v))+f_1(v)F_2(v)+f_2(v)F_1(v)\Big]\,dv$$
$$=\int_c^d -\tfrac{1}{2}v(f_1(v)-f_2(v))(F_1(v)-F_2(v))\,dv$$
$$=\int_c^d \tfrac{1}{4}(F_1(v)-F_2(v))^2\,dv,$$

where the last equality is obtained by integration by parts (noting that $F_1(v)-F_2(v)=0$ for $v=c$ and d). ◁

STEP 3. $R-\overline{R}=\int_c^d \frac{1}{4}(d\beta(v)/dv)(F_1(v)-F_2(v))^2\,dv$.

PROOF. Using the second expression of $\overline{R}$, we have that $R-\overline{R}$ can be written as

$$\int_c^d \beta(v)\Big[f_1(v)F_2(v)+f_2(v)F_1(v)-2\tfrac{1}{2}(f_1(v)+f_2(v))\cdot\tfrac{1}{2}(F_1(v)+F_2(v))\Big]\,dv$$
$$=\int_c^d -\tfrac{1}{2}\beta(v)(f_1(v)-f_2(v))(F_1(v)-F_2(v))\,dv$$
$$=\int_c^d \tfrac{1}{4}\frac{d\beta(v)}{dv}(F_1(v)-F_2(v))^2\,dv,$$

where the last equality is obtained by integration by parts (noting that $F_1(v)-F_2(v)=0$ for $v=c$ and d). ◁

Observe that $d\beta(v)/dv>0$ for all v. Hence, Proposition 1 follows from Steps 1, 2, and 3. □

PROOF OF PROPOSITION 2. We proceed in the same way as for Proposition 1, and define R as the revenue generated in the first-price auction with bidder-anonymous feedback partition, R^{SPA} as the revenue generated in the second-price auction, and $\overline{R}$ as the expected revenue generated in the second-price auction with symmetric bidders and density of valuations $\overline{f}(v)=(f_1(v)+\cdots+f_n(v))/n$. We also denote by $\beta(v)$ the equilibrium bidding strategy in the first-price auction in which the densities are $\overline{f}(v)=(f_1(v)+\cdots+f_n(v))/n$. We denote by

$$H(v)=F_1(v)\cdots F_n(v)$$

and

$$H_{\text{sym}}(v)=\overline{F}(v)^n,$$

where $F_i(v)$ is the cumulative of v according to $f_i(\cdot)$ and $\overline{F}(v)$ is the cumulative of v according to $\overline{f}(\cdot)$.

Revenue equivalence and Claim 1 imply that

$$R = \int_c^d \beta(v)H'(v)\,dv = \beta(d) - \int_c^d \beta'(v)H(v)\,dv$$

$$\overline{R} = \int_c^d \beta(v)H'_{\text{sym}}(v)\,dv = \beta(d) - \int_c^d \beta'(v)H_{\text{sym}}(v)\,dv.$$

Thus,

$$R - \overline{R} = \int_c^d \beta'(v)[H_{\text{sym}}(v) - H(v)]\,dv.$$

Given that $H_{\text{sym}}(v) \geq H(v)$ due to the concavity of $x \to \ln x$ and given that $\beta(\cdot)$ is increasing, we conclude that $R \geq \overline{R}$. Proposition 2 follows. □

Proof of Proposition 3. We start with the following observation.

Step 1. *Myerson's optimal auction can be implemented while satisfying the ex post quitting rights of the bidders in a direct truthful mechanism in which reporting the truth is a weakly dominant strategy for every bidder.*

Proof. This is easily shown by simple adaptation of the second-price auction to the optimal auction of Myerson. In the asymmetric regular case, the functions $c_i(v_i) = v_i - (1 - F_i(v_i))/f_i(v_i)$ are increasing in v_i, and the optimal auction requires allocating the object to bidder $i^* \in \arg\max_{i\in I} c_i(v_i)$ whenever $c_{i^*}(v_{i^*}) > v_s$ (and otherwise the seller should keep the object). This is achieved in a direct mechanism implementable in dominant strategy in which bidder i^* is required to pay $\max_{j\neq i}[c_i^{-1}(c_j(v_j)), c_i^{-1}(v_s)]$. It is easily checked that this payment is always less than v_{i^*} by the monotonicity of $c_i(\cdot)$. A similar construction can be achieved in general (not necessarily regular) in which intervals of valuations are treated alike.[48] ◁

The rest of the argument goes as follows. Consider a monotonic bijection ψ from $[c, d]$ into itself and let M^ψ be the mechanism obtained from the mechanism M^D identified in Step 1 as follows: in M^ψ, every bidder i submits a bid b_i and mechanism M^D is applied to the profile of announcements $(\psi(b_i))_{i=1}^n$. Clearly, M^ψ falls in the class of admissible mechanisms and reporting $\psi^{-1}(v_i)$ for bidder i with valuation v_i is a weakly dominant strategy. Besides, M^ψ achieves Myerson's optimal auction revenues and no bidder is willing to exercise his ex post quitting rights in M^ψ.

Consider now the following auction design. Format M^ψ is used with probability $1 - \varepsilon$ and the first-price auction with v_s reserve price referred to as FPA is used with probability ε. Besides, bidders get to know only the aggregate distribution of bids of all bidders across both formats. That is, we consider the bidder-anonymous and format-anonymous analogy partitions in which for all i, $\bigcup_{(j,k)}\{(j,k)\}$ forms the unique analogy class of P_i. We show that for a suitable choice of ε and ψ, this auction design generates

48One can easily perturb the format so as to make incentives strict in all cases (even in the nonregular case).

strictly more expected payoff to the seller than Myerson's optimal auction. First, we observe that the payoff generated in this auction design can be written as $(1-\varepsilon)R^{\psi}+\varepsilon R^{*}$, where R^{ψ} is the expected payoff generated in this auction design when M^{ψ} prevails and R^{*} is the corresponding expected payoff when FPA prevails. It is clear that R^{ψ} is equal to Myerson's optimal auction payoff R^{M}, since the behaviors in M^{ψ} are unaffected by the rest of the auction design given that bidders have (weakly) dominant strategies in M^{ψ}. Thus, it suffices to show that $R^{*}>R^{M}$ for suitable choices of ε and ψ.

To this end, let ψ be defined such that for all $b>v_s$,[49]

$$\prod_{i=1}^{n} F_i(\psi(b))=\left(\frac{b-\widehat{c}}{d-\widehat{c}}\right)^{nm/(n+1)}$$

for some $\widehat{c}>v_s$ and m.

In the limit case in which $\varepsilon=0$, the (perceived) optimal bid in FPA for a bidder with valuation $v>v_s$ is $\arg\max_{b>v_s}(v-b)((b-\widehat{c})/(d-\widehat{c}))^m$ as $((b-\widehat{c})/(d-\widehat{c}))^m$ represents the perceived probability that all other bidders' bids are below b. This expression is maximized at b^{opt} such that

$$b^{\text{opt}}-\widehat{c}=\frac{m}{m+1}(v-\widehat{c}).$$

Let b^{*} be such that $b^{*}-\widehat{c}=m/(m+2)(v-\widehat{c})$ and consider $\varepsilon>0$. A bidder with valuation $v>v_s$ perceives getting at most

$$(1-\varepsilon)(v-b^{*})\left(\frac{b^{*}-\widehat{c}}{d-\widehat{c}}\right)^{m}+\varepsilon(v-\widehat{c}) \tag{3}$$

by bidding $b<b^{*}$.

By bidding b^{opt}, a bidder with valuation $v>v_s$ perceives getting at least

$$(1-\varepsilon)(v-b^{\text{opt}})\left(\frac{b^{\text{opt}}-\widehat{c}}{d-\widehat{c}}\right)^{m}. \tag{4}$$

Hence, whenever (4) is larger than (3) we can be sure that a bidder with valuation v bids no less than $\widehat{c}+m/(m+2)(v-\widehat{c})$. The difference between (4) and (3) is written as

$$\Delta(v)=(1-\varepsilon)\frac{(v-\widehat{c})^{m+1}}{(d-\widehat{c})^{m}}\left[\frac{1}{m+1}\left(\frac{m}{m+1}\right)^{m}-\frac{2}{m+2}\left(\frac{m}{m+2}\right)^{m}\right]-\varepsilon(v-\widehat{c}).$$

Given that $1/(m+1)(m/(m+1))^m-2/(m+2)(m/(m+2))^m>0$, this allows us to obtain Step 2.

STEP 2. *$\forall \underline{v}>\widehat{c}$, $\forall m$, $\exists\overline{\varepsilon}>0$ such that $\forall\varepsilon<\overline{\varepsilon}$, $\forall v>\underline{v}$, $\Delta(v)>0$.*

From Step 2 and the above considerations, we infer that for all $v>\underline{v}$, $b^{\text{FPA}}(v)-\widehat{c}>m/(m+2)(v-\widehat{c})$ in the above auction design as defined by ψ and $\varepsilon<\overline{\varepsilon}$. The corresponding value of R^{*} converges to the full information revenue R^{F} as m converges to

[49] It is clear that such ψ exists and satisfies $\psi(c)=c$ and $\psi(d)=d$.

infinity and $\underline{v}$ converges to $\widehat{c}$. It follows that one can find m large enough, $\widehat{c}$ close enough to v_s, $\underline{v}$ close enough to $\widehat{c}$, and $\varepsilon > 0$ so that $R^* > R^M$. This completes the proof of the proposition. □

Proof of Proposition 4. Consider an auction design assumed to deliver an expected payoff that is ε-close to R^F. We show that this is not possible for ε small enough.

To simplify the notation, we consider the case of two symmetric bidders $i = 1, 2$ and we allow only for auction designs with format-anonymous analogy partitions. The argument easily generalizes to the n asymmetric bidder case with arbitrary analogy partitions (by restricting attention to those formats that are pooled together into one analogy class of say bidder i).

Observe that in all formats, whenever $v_j < v_s$, we must have $\beta_j^k(v_j) < v_s$ given the payment rules of the auction (and the fact that there is a tiny cost to cancelling the transaction). We further let $\gamma = \Pr(v_j < v_s, j \neq i) > 0$ and let $\Delta^k(v)$ denote the expected revenue loss incurred by the designer in format M_k when bidder i has valuation v as compared with the full information case.

Let m be large enough and let $e < \frac{1}{2}\Pr((d+v_s)/3 < v_j < (d+v_s)/2)$. Define $\underline{d} > (d+v_s)/3$ such that $e = \Pr(\underline{d} < v_i < (d+v_s)/2)$. Finally, let $f = \min(\Pr(v_i > (3d+v_s)/4), e)$ and $\eta = m\varepsilon\gamma/f$. We define

$$\Gamma = \left\{k \text{ s.t. } \exists v > \tfrac{3}{4}d \text{ and } \exists v' \in \left(\underline{d}, \tfrac{1}{2}(d+v_s)\right), \Delta^k(v) < \eta \text{ and } \Delta^k(v') < \eta\right\}.$$

Given that the auction design delivers an expected payoff that is ε-close to R^F, it is readily verified that $\sum_{k\in\Gamma} \lambda_k \geq 1 - 1/m$.

Perceived equilibrium payoff

We note that for $v > \underline{d}$, if $\Delta^k(v) < \eta$, then a bidder with valuation v should in format M_k win whenever $b_j < v_s$ (which happens with probability γ) and pay at least $v - \eta/\gamma$. By monotonicity of the payment rule, this implies that in M_k, bidder i with valuation v perceives in equilibrium to get at most

$$\frac{\eta}{\gamma} = \frac{m\varepsilon}{f}$$

(the payment when i wins, bids b_i, and $b_j > v_s$ must be at least as large as when i wins, bids b_i, and $b_j < v_s$).

We also note that $\Delta^k(v) < \eta$ implies that in M_k, bidder i with valuation v should win against some $v_j \in (\underline{d}, (d+v_s)/2)$ with probability at least $1 - \eta/(fv) = 1 - m\varepsilon\gamma/(f^2 v)$. We choose ε small enough so that this probability is no less than $\frac{1}{2}$.

Perceived equilibrium from downward deviation

One can rank the various $k \in \Gamma$ by decreasing order of $\beta^k((d+v_s)/3)$ and let $\bar{r}$ denote the maximum l such that the sum of λ_k over the first $l-1$ formats in Λ is strictly below $\frac{1}{2}\sum_{k\in\Gamma} \lambda_k$. We denote by Λ^{sup} the formats in Λ that correspond to the first $\bar{r}$ formats in this induced order.

Consider $k \in \Lambda^{\text{sup}}$ and let $v' \in (\underline{d}, (d+v_s)/2)$, $\Delta^k(v') < \eta$. Consider any $v > (3d+v_s)/4$ and let v submit a bid $b_i = \beta_i^k(v')$. Bidder i with valuation v in format M_k

must perceive to be winning with probability at least $\frac{1}{4}(1-m)\Pr(v_j < (d+v_s)/3)$,[50] and he must be paying at most $d/2$ whenever he wins.[51] Given that $v > (3d+v_s)/4$ (and thus $(3d+v_s)/4-(d+v_s)/2=(d-v_s)/4)$, overall such a deviation makes bidder i feel he can get at least $\frac{1}{2}(1-m)(d/4)\Pr(v_j < (d+v_s)/3)$ in M_k.

Given that ε can be chosen so that $m\varepsilon/f < \frac{1}{2}(1-m)(d/4)\Pr(v_j < (d+v_s)/3)$, we get a contradiction to the definition of an analogy-based expectation equilibrium (since a bidder should obviously feel that his perceived payoff obtained by following his equilibrium strategy is no less than his perceived payoff obtained by following any other strategy). □

Proof of Proposition 5. We consider the following formats M_1 and M_2, both used with probability $\frac{1}{2}$. In both formats, the good is never allocated whatever the bids, $\varphi_i^k(b)=0$ for all i, b and $k=1,2$. In format M_1, bidder 1 wins ε if $b_1=1$ and 0 otherwise. In format M_2, bidder 1 wins ε if $b_1=2$ and 0 otherwise. In format M_2, bidder 2 pays $A/2>0$ if $b_1=2$ and $b_2=1$, and receives A if $b_1=1$ and $b_2=1$, and receives nothing otherwise, i.e.,

$$\tau_2^2(b)=\begin{cases} A/2 & \text{if } (b_1,b_2)=(2,1) \\ -A & \text{if } (b_1,b_2)=(1,1) \\ 0 & \text{if } (b_1,b_2)\neq(2,1),(1,1) \end{cases}$$

and the analogy partition is the anonymous-format analogy partition.

Clearly, in this auction design, bidder 1 bids $b_1=1$ in M_1 and $b_1=2$ in M_2. Given that $\lambda_1=\lambda_2=\frac{1}{2}$ and the format-anonymous analogy partition is being used, bidder 2 believes that in M_2, bidder 1 bids $b_1=1$ or 2, each with probability $\frac{1}{2}$. Based on this belief, bidder 2 finds it optimal to bid $b_2=1$ in M_2 (because $\frac{1}{2}(A-A/2)>0$).

In such an analogy-based expectation equilibrium, the designer gets a revenue equal to $-\varepsilon$ in M_1 and $A/2-\varepsilon$ in M_2, so an overall expected revenue of $A/4-\varepsilon$. Since A can be chosen arbitrarily large, we get the desired result. □

Proof of Proposition 6. Consider the format-anonymous analogy partition in the auction design in which a share $1-\varepsilon$ of SPA is mixed with ε of the format $M(\eta,\mu)$ in which the bidder with the highest bid, say bidder 1 if $b_1>b_2$, wins the auction and pays a price

$$\tau(b_1,b_2)=(1-\eta(1+\mu b_2))b_2+\eta(1+\mu b_2)b_1.$$

(Note that as $\eta=0$, $M(\eta,\mu)$ is the second-price auction.)

[50]This is because in the format-anonymous feedback partition, all the bids $\beta_j^{k'}(v_j)$ with $v_j<(d+v_s)/3$ and $k'\in\Lambda\setminus\Lambda^{\sup}\cup\{\bar{r}\}$ must be below $\beta_i^k(v')$, and by construction $\Lambda\setminus\Lambda^{\sup}\cup\{\bar{r}\}$ has a probability at least $\frac{1}{2}(1-m)$. Moreover in M_k, i with valuation v should win against some $v'\in(\underline{d},(d+v_s)/2)$ with a probability at least $\frac{1}{2}$ (see above), and thus by the monotonicty of $\varphi_i^k(b)$ with respect to b_j, he should also win against all bids that are below $\beta_k^i(\underline{d})$ with a probability at least $\frac{1}{2}$.

[51]This is because he is mimicking type v' who never pays more than v' when he wins.

We subtract c from all bids and valuations. As ε tends to 0, in format $M(\eta,\mu)$, a bidder with valuation v chooses his bid $b(v)$ so that

$$v - b - \eta\int_0^b (1+\mu x)\,dx = 0$$

or

$$b(v) = \frac{-(1+\eta) + ((1+\eta)^2 + 2\eta\mu v)^{1/2}}{\eta\mu}$$

given that bidders expect bids to be uniformly distributed on $(0,1)$. The revenue so generated in format $M(\eta,\mu)$ can be written as

$$R(\eta,\mu) = \int_0^1\int_0^v \left\{\eta(1+\mu x)\frac{((1+\eta)^2+2\eta\mu v)^{1/2}-(1+\eta)}{\eta\mu} + (1-\eta-\eta\mu x)\frac{((1+\eta)^2+2\eta\mu x)^{1/2}-(1+\eta)}{\eta\mu}\right\}dx\,dv.$$

Simple algebra yield that $R(\eta,\mu=0)$ is identical to the revenue in the second-price auction. Moreover,

$$\frac{\partial R}{\partial\mu}(\eta,\mu=0) = \left(\frac{\varepsilon}{1+\varepsilon}\right)^3.$$

Thus, there must exist $\mu > 0$ and $\eta > 0$ sufficiently small so that $R(\eta,\mu) > 0$. The manipulative design as described above with ε sufficiently small induces a strictly higher revenue than the second-price auction. Given that the revenue in the second-price auction is the optimal revenue by Myerson's analysis, we conclude as desired (observe that for η and μ small enough, $\eta(1+\mu x) \in (0,1)$ so that $M(\eta,\mu)$ belongs to the required space of mechanisms). □

References

Athey, Susan and Philip A. Haile (2006), "Empirical models of auctions." Unpublished paper, Department of Economics, Harvard University. [186]

Cabrales, Antonio (1999), "Adaptive dynamics and the implementation problem with complete information." *Journal of Economic Theory*, 86, 159–184. [202]

Cabrales, Antonio and Roberto Serrano (2007), "Implementation in adaptive better-response dynamics." Working Paper 0708, CEMFI. [202]

Cantillon, Estelle (2008), "The effects of bidders' asymmetries on expected revenue in auctions." *Games and Economic Behavior*, 62, 1–25. [196]

Crawford, Vincent P., Tamar Kugler, Zvika Neeman, and Ady Pauzner (2009), "Behaviorally optimal auction design: Examples and observations." *Journal of European Economic Association*, 7, 377–387. [202]

Dekel, Eddie, Drew Fudenberg, and David K. Levine (2004), "Learning to play Bayesian games." *Games and Economic Behavior*, 46, 282–303. [201]

Eliaz, Kfir (2002), "Fault tolerant implementation." *Review of Economic Studies*, 69, 589–610. [202]

Eliaz, Kfir and Ran Spiegler (2007), "A mechanism-design approach to speculative trade." *Econometrica*, 75, 875–884. [202]

Esponda, Ignacio (2008a), "Information feedback in first price auctions." *RAND Journal of Economics*, 39, 491–508. [187, 201]

Esponda, Ignacio (2008b), "Behavioral equilibrium in economies with adverse selection." *American Economic Review*, 98, 1269–1291. [202]

Ettinger, David and Philippe Jehiel (2010), "A theory of deception." *American Economic Journal: Microeconomics*, 2, 1–20. [199]

Eyster, Erik and Matthew Rabin (2005), "Cursed equilibrium." *Econometrica*, 73, 1623–1672. [202]

Fudenberg, Drew and David K. Levine (1998), *The Theory of Learning in Games*. MIT Press, Cambridge, Massachusetts. [185, 200]

Güth, Werner, Radosveta Ivanova-Stenzel, and Elmar Wolfstetter (2005), "Bidding behavior in asymmetric auctions: An experimental study." *European Economic Review*, 49, 1891–1913. [196]

Harsanyi, John C. (1995), "Games with incomplete information." *American Economic Review*, 85, 291–303. [185]

Huck, Steffen, Philippe Jehiel, and Tom Rutter (2011), "Learning spillovers and analogy-based expectations: A multi-game experiment." *Games and Economic Behavior*, 71, 351–365. [201]

Jehiel, Philippe (2005), "Analogy-based expectation equilibrium." *Journal of Economic Theory*, 123, 81–104. [185, 186, 191, 199, 202]

Jehiel, Philippe and Frédéric Koessler (2008), "Revisiting games of incomplete information with analogy-based expectations." *Games and Economic Behavior*, 62, 533–557. [191, 199, 202]

Lehrer, Ehud (2008), "Partially-specified probabilities: Decisions and games." Unpublished paper, School of Mathematical Sciences, Tel Aviv University. [204]

Maskin, Eric S. and John G. Riley (2000), "Asymmetric auctions." *Review of Economic Studies*, 67, 413–438. [188]

Matsushima, Hitoshi (2008), "Detail-free mechanism design in twice iterative dominance: Large economies." *Journal of Economic Theory*, 141, 134–151. [202]

McAfee, R. Preston and John McMillan (1987), "Auctions with a stochastic number of bidders." *Journal of Economic Theory*, 43, 1–19. [206]

Milgrom, Paul R. (2004), *Putting Auction Theory to Work*. Cambridge University Press, Cambridge. [188]

Morris, Stephen (1994), "Trade with heterogeneous prior beliefs and asymmetric information." *Econometrica*, 62, 1327–1347. [203]

Myerson, Roger B. (1981), "Optimal auction design." *Mathematics of Operations Research*, 6, 58–73. [188, 191, 193, 201]

Reynolds, Stanley S. and John Wooders (2009), "Auctions with a buy price." *Economic Theory*, 38, 9–39. [207]

Riley, John G. and William F. Samuelson (1981), "Optimal auctions." *American Economic Review*, 71, 381–392. [188]

Simon, Herbert A. (1955), "A behavioral model of rational choice." *Quarterly Journal of Economics*, 69, 99–118. [202]

Submitted 2009-12-2. Final version accepted 2010-8-2. Available online 2010-8-2.