Contract Terms Monotonicity in Matching Markets*

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Abstract

We study doctor–hospital matching markets with contracts, in which hospitals can offer a range of contract terms to doctors. We show that introducing new terms can reduce doctors' welfare, whereas withdrawing terms may generate a Pareto improvement. We establish the preference domains under which welfare is preserved and show that only *agent-lexicographic preferences* for all agents ensure that no doctor is worse off when additional terms are introduced. Since this condition is rarely satisfied in practice, our results imply that most real-world markets are vulnerable to welfare losses when the set of contract terms expands.

Keywords: Matching with contracts; Stability; Contract terms; Welfare; Agent-lexicographic preferences.

JEL Classification: C78, D47, D71, D86.

1 Introduction

Contract terms play a crucial role in many matching markets. In labor markets, workers are assigned shifts and earn salaries; doctors are assigned to specialties within hospitals; and cadets are allocated to military branches. Some terms

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are regulated—such as minimum wages, pay scales, or legal limits on working hours—while others are determined at the discretion of hospitals or firms, for example when they choose which specialties to offer, how to structure working hours, or whether to allow remote work. In both decentralized and centralized markets, hospitals and firms may introduce new contract terms either autonomously or when institutional or regulatory changes authorize additional options—for instance, newly permissible shift structures, part-time positions, or night-work arrangements. These expansions increase contractual flexibility and directly shape participants' welfare. Despite the frequency and policy relevance of such changes, their welfare consequences remain poorly understood. This raises a fundamental question: does expanding the range of available terms always improve welfare?

For concreteness, we adopt the terminology of doctors and hospitals, although the results apply to other matching environments such as school choice and housing.[1] Across markets—whether organized through centralized clearinghouses such as the National Resident Matching Program (NRMP) or through decentralized institutions—the set of available contract terms is shaped by legal, institutional, and organizational factors. In the NRMP, for example, the clearinghouse determines which contract dimensions are standardized across hospitals.[2] In decentralized labor markets, hospitals choose which terms to offer—such as new specialties, non-wage amenities, or compensation schemes—subject to similar constraints.

Among the various contract dimensions, salary has attracted particular attention. An antitrust case investigated residency pay practices and, although ultimately dismissed, reinforced the standardization of salaries across programs and specialties.[3] Such constraints are prevalent in many matching markets, limit the scope for salary adjustments, and may prevent the existence of competitive equilibria.

To analyze the effects of adding contract terms when salaries or other dimensions

[1] In school choice, contract terms may represent tuition fees or students' chosen academic pathways, whereas in housing markets they may reflect factors such as the rent amount.

[2] The NRMP processes over 48,000 applications for 38,000 positions annually across more than 60 subspecialties, which may naturally be modeled as contract terms (https://www.nrmp.org/).

[3] See Bulow and Levin (2006), Kojima (2007) and Niederle (2007) for discussion. Residency salaries, typically around $40,000 per year, were criticized as compressed across programs and specialties relative to those of more senior physicians, despite working hours of up to 80 hours per week.

are constrained, we rely on the concept of stability and adopt the many-to-one matching with contracts model (Hatfield and Milgrom 2005). Stability yields allocations that are immune to mutually beneficial deviations and is therefore a natural equilibrium notion in environments where contracts are formed through bilateral negotiation rather than price competition. Although introducing new contract terms expands doctors' opportunities, additional terms may intensify competition for desirable positions. As a result, while increased contractual flexibility can improve doctors' welfare,[4] it may also reduce it. Using an illustrative example in which additional terms represent higher salary levels, we first show that doctors' welfare may decrease even when the new terms are introduced simultaneously at all hospitals and lie on the Pareto frontier. We then show that the same effect can arise when only a single hospital introduces an additional term. This demonstrates that the potential welfare loss does not stem from the centralized or coordinated nature of the intervention, but from the expansion of the feasible set of allocations. The intuition is that new terms enlarge the set of feasible allocations, potentially blocking previously stable ones and forcing some doctors to accept less-preferred contracts to secure placement.

Our first theorem establishes the connection between stable allocations before and after the addition of contract terms. Specifically, it shows that if an allocation is stable after new terms are introduced and was feasible—that is, the terms used in its contracts were already available at the corresponding hospitals—then it was also stable prior to the introduction of these terms. This result also implies that withdrawing unused terms from a stable allocation preserves stability. At the doctor-optimal stable allocation, the resulting allocation is either a Pareto improvement for doctors or identical to the original one. However, we identify two limitations to this approach. First, the Pareto improvement is not always possible. Second, when hospitals' preferences satisfy the classical law of aggregate demand, it is impossible to strictly improve all doctors' allocations, and therefore adding terms cannot strictly reduce all doctors' welfare.

We then analyze conditions on agents' preferences that prevent welfare reductions

[4] Adding contract terms has a different impact than adding agents. Crawford (1991) shows that introducing a new hospital weakly improves doctors' welfare at the doctor-optimal stable allocation, with a strict improvement for some doctors if the new hospital is assigned.

when new terms are introduced, aiming to identify which markets are susceptible to this phenomenon. We show that traditional approaches based on *acyclic* preference structures (Ergin 2002) do not rule out welfare reductions. Instead, our results indicate that only *agent-lexicographic preferences* (Pakzad-Hurson 2023), imposed on both sides of the market, prevent welfare losses when new terms are added. Under such preferences, the primary criterion for ranking contracts is the identity of the hospital or doctor, while contract terms serve only as a secondary criterion. This condition defines the maximal domain for which welfare reductions are prevented: if any agent—doctor or hospital—does not have agent-lexicographic preferences, a reduction in welfare can occur. If hospitals' preferences are agent-lexicographic but doctors' are not, additional terms may attract a doctor who is preferred by a hospital to one of its previously employed doctors, thereby reducing the welfare of the latter. Conversely, if doctors' preferences are agent-lexicographic but hospitals' are not, hospitals may introduce new terms they consider preferable and recruit doctors willing to accept any contract for employment at that hospital, again reducing the welfare of those previously assigned. In both cases, agents' preferences over the newly added terms generate competition, leading to welfare reductions. In practice, however, agent-lexicographic preferences are rarely observed, implying that almost any real-world market is vulnerable to welfare reductions when contract terms are added. To model more realistic preferences, we consider the domains of *common* and *polarized* preferences. Under common preferences, all doctors (respectively hospitals) share the same ranking over contract terms. Polarization further requires that whenever doctors prefer one term to another, hospitals prefer the latter to the former. Salary negotiations provide a direct application of this domain, with doctors favoring higher salaries at a given hospital while hospitals prefer lower salaries for a given doctor. We then identify conditions ensuring that no doctor is worse off when additional terms are introduced for a hospital, as well as conditions under which at least one doctor is strictly better off. For the latter, three conditions must hold simultaneously: at least one of the added terms is used; the hospital introducing the new terms continues to employ the same set of doctors; and doctors prefer the added terms to those previously available.

To assess the impact of adding contract terms on doctors' welfare in practice, we conduct simulations. When preferences over contract terms are random—for example, reflecting differences in shifts or medical specialties—and the number of doctors exceeds available positions, adding terms results in roughly equal shares of doctors being better or worse off, affecting approximately 9–21% of doctors. When preferences over terms are polarized, the share of doctors who are worse off is smaller than those who are better off, but still substantial. These simulations highlight the need to carefully regulate the introduction of new terms.

Finally, we examine the robustness of our results by considering the competitive equilibrium as a solution concept, assuming fully flexible salaries. Within the class of valuation profiles in which hospital valuations are *additively separable*, we establish the maximal domain under which adding contract terms never reduces doctors' welfare at the doctor-optimal competitive equilibrium. Specifically, we show that welfare preservation requires the agent-valuation profile to exhibit *common marginal term valuations*, that is, the marginal value of substituting one contract term for another must be identical across all doctors at a given hospital, and symmetrically across all doctors from that hospital's perspective. Such uniformity is rarely satisfied in practice, since terms often reflect personal circumstances. A parking space or a childcare slot, for instance, generates a strictly positive marginal valuation only for doctors who own a car or have young children, while others remain indifferent, so that doctors' marginal valuations naturally differ across terms. Hence, almost all markets remain vulnerable to welfare losses when new contract terms are introduced.

Related Literature

The economics literature on matching markets focuses on two solution concepts: stability and competitive equilibrium (Gale and Shapley 1962; Shapley and Shubik 1971), extended to contract terms and many-to-one settings by Crawford and Knoer (1981), Kelso and Crawford (1982), Fleiner (2003), and Hatfield and Milgrom (2005). These two solution concepts are similar in the allocations they produce. In particular, when contracts are substitutes for hospitals, Echenique (2012) shows that bargaining over contracts can be embedded into a flexible-salary model of bargaining over salaries, and Schlegel (2015) further characterizes the conditions under which such an embedding is possible.

When salaries are institutionally constrained, as in the NRMP, market-clearing prices may not exist (Kamecke 1998), making stability the natural solution concept. In a one-to-one setting, Bulow and Levin (2006) show that impersonal salary-setting compresses workers' wages below any competitive equilibrium outcome, though Kojima (2007) demonstrates that this effect does not extend to many-to-one markets. Bulow and Levin (2006) also show that the resulting allocation is inefficient relative to the competitive equilibrium. More recently, Jungbauer (2026) shows that strategic salary posting generally does not lead to efficient outcomes in many-to-one settings. To restore efficiency, Niederle (2007) and Crawford (2008) propose introducing multiple salary levels to approximate the competitive equilibrium—which, in our framework, corresponds to adding contract terms. We show, however, that such additions do not necessarily improve welfare, even when higher salaries are offered simultaneously across all hospitals, highlighting the importance of contract design more broadly.

Beyond salaries, contract terms arise in many settings—enlistment duration in the U.S. Army (Sönmez 2013; Sönmez and Switzer 2013), hospital department assignments (Hatfield and Kominers 2015), and airline upgrades (Kominers and Sönmez 2016). While these studies provide important insights into agents' welfare, they do not address the design of contract terms. Hatfield and Kominers (2017) emphasize the role of contract design in many-to-many matching with contracts; in contrast, we show that limiting the set of available terms can improve doctors' welfare within stable allocations. This perspective is particularly relevant in environments where each hospital is required to offer the same terms to all doctors, such as markets in which discrimination is prohibited. In such settings, a hospital cannot offer different salaries or working conditions to different doctors, yet welfare losses can still occur when new terms are introduced, as illustrated in Example 7 (Appendix A).

Our paper also contributes to the literature on preference structure and welfare. Ergin (2002) shows that acyclic preferences guarantee stable and Pareto efficient assignments, a result that Pakzad-Hurson (2023) extends to markets with contracts by introducing agent-lexicographic preferences on one side of the market. We show that preventing welfare reductions when terms are added requires imposing this condition on both sides—a substantially stronger requirement that limits the scope for policy intervention, since it cannot be achieved through priority design alone. Nei and Pakzad-Hurson (2021) study the college admissions market, where contract terms correspond to available majors, and show that when colleges adopt student-lexicographic preferences, students' welfare improves because they retain flexibility to choose among terms once admitted. However, this reduces colleges' control, incentivizing them to favor certain contract terms over others—a design choice commonly adopted, illustrating that the condition of agent-lexicographic preferences is rarely satisfied.

Finally, this paper contributes to the welfare literature in matching markets. Chambers and Yenmez (2017), Afacan (2017), and Yenmez (2018) analyze how choice functions and acceptable contract sets affect stable allocations. In contrast, we examine how the availability of contract terms affects doctors' welfare, without altering hospitals' choice functions. Our approach also relates to the notion of consent introduced by Kesten (2010), who considers restricting access to improve assignments in school choice. However, whereas Kesten (2010) considers a Pareto efficient but not necessarily stable assignment, we maintain the stability of allocations.

The paper is organized as follows. Section 2 introduces the model and a motivating example. Section 3 analyzes the impact of adding terms on stable allocations and doctors' welfare. Section 4 identifies conditions on preferences that prevent welfare reductions. Section 5 presents simulations on how additional terms affect doctors' welfare. Section 6 examines the implications for competitive equilibria. Section 7 concludes. Additional examples on welfare reductions are provided in Appendix A, and all proofs are collected in Appendix B.

2 Model

2.1 Allocation Problem

There are finite sets $D = \{d_1, d_2, ..., d_n\}$ of *doctors*, $H = \{h_1, h_2, ..., h_m\}$ of *hospitals*, and $\mathcal{T} = \{t_1, t_2, ..., t_\ell\}$ of *terms*. The set of *contracts* is $\mathcal{X} = D \times H \times \mathcal{T}$.

Each contract $x \in \mathcal{X}$ involves a doctor $x_d \in D$, a hospital $x_h \in H$, and a term $x_t \in \mathcal{T}$. The null contract, which indicates that the doctor or hospital has no contract, is denoted by $\varnothing$. For $X \subseteq \mathcal{X}$, let $X_d \equiv \{x \in X : x_d = d\}$ and $X_h \equiv \{x \in X : x_h = h\}$ denote the contracts in X involving doctor d and hospital h, respectively. Each hospital $h \in H$ has a *capacity* $q_h \in \mathbb{N}$, and $q \equiv (q_h)_{h\in H}$ denotes the *vector of capacities*. A set of contracts $X \subseteq \mathcal{X}$ is an *allocation* if each doctor $d \in D$ is involved in at most one contract, $|X_d| \leq 1$, and each hospital $h \in H$ is involved in at most q_h contracts, $|X_h| \leq q_h$. It will often be useful to restrict attention to a subset of terms offered by each hospital. To this end, for each hospital h, let $T_h \subseteq \mathcal{T}$ denote the set of *available terms*. To ensure that each hospital can form contracts, we assume $T_h \neq \emptyset$ for each $h \in H$.

Each doctor $d \in D$ has *strict preference* $\succ_d$ over $\mathcal{X}_d \cup \{\varnothing\}$, with $\succ_D \equiv (\succ_d)_{d\in D}$ denoting the *preference profile of doctors*. A contract is *acceptable* if it is strictly preferred to the null contract $\varnothing$ and *unacceptable* otherwise. Doctor d's *choice function* is $C_d(X) \equiv \max_{\succ_d}[\{x \in X_d : x_t \in T_{x_h}\} \cup \{\varnothing\}]$.[5] Let $C_D(X) \equiv \bigcup_{d\in D} C_d(X)$ be the set of contracts chosen from X by doctors.

Each hospital $h \in H$ has a strict preference $\succ_h$ over sets $X \in 2^{\mathcal{X}_h} \cup \{\varnothing\}$, with $\succ_H \equiv (\succ_h)_{h\in H}$ denoting the *preference profile of hospitals*. Throughout, we assume that any set of contracts that violates a hospital's capacity or includes multiple contracts for the same doctor is unacceptable. Formally, for each $X \subseteq \mathcal{X}_h$, if $|X| > q_h$ or there exists $d \in D$ with $|X_d| > 1$, then $\varnothing \succ_h X$. Hospital h's *choice function* is $C_h(X) \equiv \max_{\succ_h}[\{X' \subseteq X_h : x_t \in T_{x_h} \text{ for each } x \in X'\} \cup \{\varnothing\}]$, and $C_H(X) \equiv \bigcup_{h\in H} C_h(X)$ collects all contracts chosen by hospitals from X.

A *problem* is a tuple $(D, H, T, q, \succ_D, \succ_H)$. Let Π be the *set of all problems*.[6] We fix $D, H, q, \succ_D$ and $\succ_H$ throughout the paper and denote a problem by $\pi(T)$. An allocation X is *feasible* if for each $x \in X$ we have $x_t \in T_{x_h}$. Let $\mathcal{F}(\pi(T))$ denote the *set of feasible allocations* for problem $\pi(T)$.

We can now introduce the notion of *stability* for allocations.

[5] We use the notation $\max_{\succ_d}$ to indicate maximization with respect to the preferences of doctor d.

[6] Strictly speaking, Π is a proper class, since arbitrary sets may serve as labels for doctors, hospitals, or terms. This issue disappears if, without loss of generality, all agents and terms are indexed by natural numbers, in which case the collection becomes a set.

Definition 1. An allocation $X \subseteq \mathcal{X}$ is *stable* if

(i) $C_D(X) = C_H(X) = X$,

(ii) there exists no hospital h and set of contracts $Y \neq C_h(X)$ such that $Y = C_h(X \cup Y) \subseteq C_D(X \cup Y)$.

When (ii) is violated by some Y, we say that Y *blocks* X.[7] We denote by $S(\pi(T))$ the *set of stable allocations* for problem $\pi(T)$.

2.2 Substitutes Condition

In this paper, we assume that contracts are *substitutes* for hospitals.

Definition 2 (Hatfield and Milgrom 2005)**.** Contracts are *substitutes* for h if there do not exist contracts $x, x' \in \mathcal{X}$ and a set of contracts $X \subseteq \mathcal{X}$ such that $x' \notin C_h(X \cup \{x'\})$ and $x' \in C_h(X \cup \{x, x'\})$.

A doctor's strict preference $\succ_d$ over contracts induces a weak preference $\succeq_d$ over allocations such that $X_d \succeq_d Y_d$ if and only if $X_d \succ_d Y_d$ or $X_d = Y_d$. Similarly, each hospital's strict preference $\succ_h$ over sets of contracts induces a weak preference relation $\succeq_h$ over allocations such that $X_h \succeq_h Y_h$ if and only if $X_h \succ_h Y_h$ or $X_h = Y_h$. An allocation $X \subseteq \mathcal{X}$ *Pareto dominates* an allocation $Y \subseteq \mathcal{X}$ if for each doctor $d \in D, X_d \succeq_d Y_d$ and for at least one doctor $d' \in D, X_{d'} \succ_{d'} Y_{d'}$. An allocation is *Pareto efficient* if it is not Pareto dominated by any other allocation. When contracts are substitutes for hospitals, at least one stable allocation exists, and in particular, there exists an allocation that is unanimously preferred by all doctors. Formally, given a problem $\pi(T)$, an allocation $\overline{X} \in S(\pi(T))$ is the *doctor-optimal stable allocation* if every doctor weakly prefers $\overline{X}$ to all other stable allocations. Moreover, the substitutes condition guarantees the monotonicity of the doctor-proposing *cumulative offer process* (COP hereafter),[8] thereby making renegotiation unnecessary.[9]

[7] Since $Y = C_h(X \cup Y)$, it follows that $|Y| \leq q_h$ and $Y \in \mathcal{F}(\pi(T))$.

[8] The COP, introduced by Hatfield and Milgrom (2005), is formally defined in Appendix B.5.

[9] Note that since we consider strict preferences, the *irrelevance of rejected contracts* condition is not required (see Aygün and Sönmez 2013).

Theorem 0 (Theorem 3 of Hatfield and Milgrom 2005)**.** Suppose contracts are substitutes for hospitals. Then, for a given vector of terms T, $S(\pi(T))$ is non-empty and forms a lattice. In addition, the COP generates the doctor-optimal stable allocation.

2.3 Motivating Example with Salaries

Salaries can be naturally modeled as contract terms. Echenique (2012) demonstrates that any matching with contracts problem can be embedded in a matching with salaries framework when contracts are substitutes for hospitals. In the case of salaries, the Pareto frontier of contracts is one-dimensional: if a term benefits a doctor, it necessarily becomes less favorable for the hospital. We present an example in which the added term corresponds to a higher salary, which is generally considered more favorable to doctors.[10]

Example 1. Consider a problem where $D = \{d_1, d_2, d_3\}$, $H = \{h_1, h_2\}$, terms are salaries with $\mathcal{T} = \{100, 110\}$, and each hospital has a capacity of one. To illustrate the effect of adding contract terms, we consider two vectors of available terms. Let T' denote the initial vector of terms, defined by $T' = (T'_{h_1} = \{100\}, T'_{h_2} = \{100\})$, and let T denote the vector of terms after additional terms are introduced, defined as $T = (T_{h_1} = \{100, 110\}, T_{h_2} = \{100, 110\})$. Contracts are substitutes for hospitals. For simplicity, we list only contracts that are mutually acceptable to both doctors and hospitals, and we restrict attention to allocations. With a slight abuse of notation, since each hospital has a capacity of one, we report hospitals' preferences over individual contracts rather than over sets of contracts.

Doctors:

$$d_1 : (d_1, h_2, 110) \succ_{d_1} (d_1, h_1, 110) \succ_{d_1} (d_1, h_2, 100) \succ_{d_1} (d_1, h_1, 100),$$
$$d_2 : (d_2, h_1, 110) \succ_{d_2} (d_2, h_1, 100) \succ_{d_2} (d_2, h_2, 110) \succ_{d_2} (d_2, h_2, 100),$$
$$d_3 : (d_3, h_1, 110).$$

Hospitals:

[10] In practice, salary increases can reflect adjustments in hospitals' budgets driven by competitive labor markets or policy interventions. For instance, U.S. hospitals often increase compensation to attract or retain doctors when facing competition from private practice groups or in response to changes in reimbursement schemes (Gawande 2009; Clemens and Gottlieb 2014).

$$h_1 : (d_1, h_1, 100) \succ_{h_1} (d_3, h_1, 110) \succ_{h_1} (d_1, h_1, 110) \succ_{h_1} (d_2, h_1, 100) \succ_{h_1} (d_2, h_1, 110),$$
$$h_2 : (d_2, h_2, 100) \succ_{h_2} (d_2, h_2, 110) \succ_{h_2} (d_1, h_2, 100) \succ_{h_2} (d_1, h_2, 110).$$

Doctor d_3 accepts employment only under the conditions specified in contract $(d_3, h_1, 110)$; otherwise, she prefers the outside option, which may correspond to working in a private clinic, i.e., the null contract. The doctor-optimal stable allocation of $\pi(T')$ is

$$\overline{X}' = \{(d_1, h_2, 100), (d_2, h_1, 100), \varnothing\}.$$

Now suppose that both hospitals offer an additional salary of 110, corresponding to the problem $\pi(T)$. The new doctor-optimal stable allocation is

$$\overline{X} = \{(d_1, h_1, 100), (d_2, h_2, 110), \varnothing\}.$$

Despite the introduction of higher salaries, doctors d_1 and d_2 are worse off, while d_3 remains assigned to the null contract; no doctor is better off. Although d_2 receives a salary of 110, she would strictly prefer employment at h_1 to h_2, regardless of the salary level. The reduction in welfare can be understood through the sequence of negotiations, in which doctors propose contracts. When hospital h_1 offers a salary of 110, both d_2 and d_3 apply for contracts involving this salary, but h_1 can hire only one doctor. Since h_1 strictly prefers the contract involving d_3 to the contracts involving d_2 at any salary, d_2 next proposes a contract at h_2. This generates competition between d_1 and d_2 at h_2, and because h_2 prefers contracts involving d_2 to those involving d_1, the contract proposed by d_1 is rejected. Doctor d_1 then applies to h_1, entering into competition with d_3. To obtain employment, d_1 must accept a contract with a lower salary.[11]

Example 1 has two important implications. First, even when a term is added simultaneously across all hospitals—for instance, through a centralized clearinghouse that expands the set of available contract terms—it may decrease the welfare of some doctors without improving that of any others. Moreover, the added term in this example is strictly preferred by doctors to those already available, offering

[11] This interpretation relates to the concept of the *reserve army of labour*, introduced by Engels and further theorized by Marx (1867).

them a higher salary. Salaries thus induce *common* preferences among doctors.[12] If the added terms were, for instance, the option to work part-time or specific work shifts, a similar effect could occur, reducing doctors' welfare.

Second, although the added term is used in the contract $(d_2, h_2, 110)$, d_2 is worse off. The increase in the salary available at h_2 did not attract d_2; rather, it generated competition that led d_2 to be reassigned to another hospital. This contrasts with the addition of agents to the market. When new agents enter a market, those assigned to them in a stable allocation are strictly better off (see Crawford 1991).

Welfare reductions may also arise when additional terms are offered only by a subset of hospitals—for example, in decentralized markets or when the clearinghouse does not regulate that dimension. To illustrate, if the new term is offered exclusively at h_1, the vector of terms is given by $T'' = (T''_{h_1} = \{100, 110\}, T''_{h_2} = \{100\})$, and the doctor-optimal stable allocation is

$$\overline{X}'' = \{(d_1, h_1, 100), (d_2, h_2, 100), \varnothing\}.$$

By offering a salary of 110, hospital h_1 signals that it provides higher compensation than other hospitals. This, however, reduces doctors' welfare while allowing the hospital to achieve a more favorable allocation. To formally analyze this phenomenon, we introduce sub-problems.

2.4 Sub-Problems and Preliminary Result

Given a problem $\pi(T) \in \Pi$, a *sub-problem* of $\pi(T)$, denoted, $\pi(T')$ with an apostrophe on the vector of terms, is obtained by reducing the set of available terms for at least one hospital: $\pi(T') \in \Pi$ is a sub-problem of $\pi(T)$ if $\pi(T) \equiv (D, H, T, q, \succ_D, \succ_H)$ and $\pi(T') \equiv (D, H, T', q, \succ_D, \succ_H)$ with $T' \subset T$, where $T'_h \neq \emptyset$ for each $h \in H$. Let $\tilde{\Pi}(\pi(T))$ denote the *set of sub-problems* of $\pi(T)$ and $\overline{X}'$ the doctor-optimal stable allocation of $\pi(T')$.

When the set of available terms for a hospital h is reduced while all other hospitals retain their original sets of terms, we represent this sub-problem as $\pi(T'_h, T_{-h}) \in \tilde{\Pi}(\pi(T))$. Conversely, we may consider cases in which the set of available terms for hospital h is expanded. In this case, we define the corresponding sub-problem

[12] We study common preferences in greater detail in Section 4.2.

as $\pi(T')$, where only hospital h experiences an expansion in its set of available terms, i.e., $\pi(T) \equiv \pi(T_h, T'_{-h})$ with $T'_h \subset T_h$.

Proposition 1 formalizes the observations of Example 1.

Proposition 1. There exist problems $\pi(T) \in \Pi$ and sub-problems $\pi(T') \in \tilde{\Pi}(\pi(T))$ such that:

(i) There exists a non-empty subset of doctors $D' \subset D$ such that, for each $d \in D'$, $\overline{X}'_d \succ_d \overline{X}_d$. Moreover, whenever $D' \neq \emptyset$, there exists a non-empty subset of hospitals $H' \subseteq H$ such that, for each $h \in H'$, $\overline{X}_h \succ_h \overline{X}'_h$.

(ii) There exists a hospital $h \in H$, a term $t \in T_h \setminus T'_h$, and a contract $x \in \overline{X}_h$ such that $x_t = t$ and $\overline{X}'_{x_d} \succ_{x_d} \overline{X}_{x_d}$.

Proposition 1 establishes that the introduction of additional terms can (i) reduce the welfare of some doctors, (ii) even when one of the added terms is used in the resulting allocation. Moreover, whenever at least one doctor is worse off, at least one hospital strictly prefers the new doctor-optimal stable allocation. In such cases, the doctors who experience welfare losses were initially employed by hospitals in H', whose contracts are replaced by contracts more favorable to those hospitals, potentially at the expense of doctors' welfare.

3 Stability and Welfare

To study the impact of adding contract terms on doctors' welfare, we first examine how the set of stable allocations is affected.

3.1 Set of Stable Allocations

The addition of new terms expands the set of feasible allocations. Such allocations can then block the doctor-optimal stable allocation of the original sub-problem. In Example 1, when a salary of 110 becomes available at h_1,

- $C_{d_3}(\{(d_3, h_1, 110), \overline{X}'_{d_3}\}) = \{(d_3, h_1, 110)\}$, and
- $C_{h_1}(\{(d_3, h_1, 110), \overline{X}'_{h_1}\}) = \{(d_3, h_1, 110)\}$.

If a previously stable allocation becomes unstable after expanding the set of available terms, at least one contract involving one of the new terms must participate in a blocking allocation.[13] Consequently, adding terms can reduce the set of stable

[13] Lemma 2 formalizes and proves this statement for any stable allocation.

allocations. Conversely, adding new terms may also generate new stable allocations. For instance, in Example 1, adding salary 110 only at h_2 leaves $\overline{X}'$ stable and allows d_1 to obtain a higher salary.

Our first main result states that any allocation that is stable in the problem $\pi(T)$ remains stable in each sub-problem of $\pi(T)$ in which the allocation is feasible.[14]

Theorem 1. For any problem $\pi(T) \in \Pi$,

(i) Suppose that $X \in S(\pi(T))$. For each $\pi(T') \in \tilde{\Pi}(\pi(T))$, if $X \in \mathcal{F}(\pi(T'))$, then $X \in S(\pi(T'))$.

(ii) For any $\pi(T'), \pi(T'') \in \tilde{\Pi}(\pi(T))$, it holds that $S(\pi(T')) \cap S(\pi(T'')) \subseteq S(\pi(T' \cup T''))$.

The converse of Theorem 1(i) is false.[15] Theorem 1(ii) shows that, given two sub-problems $\pi(T')$ and $\pi(T'')$ of a problem $\pi(T)$, any allocation that is stable in both sub-problems remains stable when the set of available terms is expanded to their union for hospitals. While this condition is restrictive, it is necessary to guarantee stability of allocations when hospitals' sets of contract terms are expanded. An additional result on the structure of stable allocations is presented in Online Appendix OA.1.

3.2 Reduction of Available Terms and Pareto Improvement

From Theorem 1, it follows that to preserve the stability of an allocation when the set of available terms is reduced, it is sufficient to maintain its feasibility. Hence, if an allocation is stable and some terms are *not used*, these terms can be withdrawn from the set of available terms without affecting stability. Formally, given an allocation X, a term t is not used at hospital h if there exists no contract

[14]This is related to Proposition 3 of Hatfield and Kominers (2017). While Hatfield and Kominers (2017) consider a subset of contracts, we consider a subset of available terms to guarantee the stability of an allocation. The advantage of our approach is that we can identify the vector of terms needed for the stability of X, which is absent from their formulations.

[15]Consider $t \notin T'_h$ such that, for each allocation $X \in S(\pi(T'))$, we have $(d,h,t) \succ_h X_h$ and $(d,h,t) \succ_d X_d$. Now consider T such that $t \in T_h$ and $T' \subset T$. An allocation with the contract $x = (d,h,t)$ blocks X.

$x \in X$ such that $x_h = h$ and $x_t = t$. We now apply this reasoning to doctor-optimal stable allocations.[16] By withdrawing terms, allocations that could block other allocations are no longer feasible, which can enable a Pareto improvement for doctors. In Example 1, the salary of 110 is not used at h_1 in $\overline{X}$. Withdrawing this term yields a Pareto improvement: d_1 receives a salary of 110 at h_2, and d_2 is employed at h_1 with a salary of 100. The following corollary formalizes this point.

Corollary 1. For any problem $\pi(T) \in \Pi$, suppose there exists $h \in H$ such that h has at least one term not used in the allocation $\overline{X}$. Then, there exists $T'_h \subset T_h$ such that $\pi(T'_h, T_{-h}) \in \tilde{\Pi}(\pi(T))$, and either $\overline{X}'$ Pareto dominates $\overline{X}$, or $\overline{X}' = \overline{X}$.

Corollary 1 implies that restricting available terms can improve doctors' welfare. This parallels Kesten (2010), who shows that withdrawing student applications can improve welfare in school choice. Kesten (2010) suggests obtaining students' consent to withdraw applications, thereby improving the welfare of other students. In the presence of contracts, this corresponds to limiting access to certain terms—such as remote work, shifts, or higher salaries—for specific doctors. Differential access to such terms can generate conflicts among doctors within hospitals, forming the basis of the general argument for stability. While Kesten (2010) aims to achieve Pareto efficiency by withdrawing applications, our approach focuses on improving doctors' welfare while maintaining stability.

We identify two limitations of withdrawing terms. First, Pareto improvements are not always guaranteed and may require restricting specific terms. In Example 1, the term 100 at hospital h_2 is not used in the allocation $\overline{X}$; however, withdrawing it does not lead to a Pareto improvement. Moreover, the next example illustrates that achieving a Pareto efficient allocation solely by withdrawing available terms is not always possible.

Example 2. Consider a problem where $D = \{d_1, d_2\}$, $H = \{h_1, h_2\}$, $\mathcal{T} = \{t, t'\}$, and each hospital has a capacity of one. The vector of terms is given by $T = (T_{h_1} = \{t, t'\}, T_{h_2} = \{t\})$. Contracts are substitutes for hospitals, and preferences are given as follows:

[16] This observation can be reformulated using the property of *irrelevance of unused terms* in a stable allocation. Hirata et al. (2023) define the *irrelevance of unchosen contracts* by adding contracts available to one doctor.

Doctors:

$$d_1 : (d_1, h_1, t) \succ_{d_1} (d_1, h_1, t'), \quad d_2 : (d_2, h_1, t) \succ_{d_2} (d_2, h_2, t).$$

Hospitals:

$$h_1 : (d_1, h_1, t') \succ_{h_1} (d_2, h_1, t) \succ_{h_1} (d_1, h_1, t), \quad h_2 : (d_2, h_2, t).$$

Consider the doctor-optimal stable allocation and an allocation X' defined by

$$\overline{X} = \{(d_1, h_1, t'), (d_2, h_2, t)\}, \quad X' = \{(d_1, h_1, t), (d_2, h_2, t)\}.$$

Although X' is the unique allocation that Pareto dominates $\overline{X}$ and is Pareto efficient, there is no $T' \subset T$ such that $X' \in S(\pi(T'))$. The term to withdraw is t for hospital h_1, but its withdrawal would prevent the feasibility of allocation X', as doctor d_1 is assigned to h_1 with term t. Achieving feasibility would require introducing doctor-specific restrictions on access to terms. However, such adjustments are beyond the scope of our analysis.

The second limitation arises under the commonly imposed law of aggregate demand, which requires that hospitals do not reduce the number of contracts they choose when the set of contracts expands.

Definition 3 (Hatfield and Milgrom 2005)**.** The preferences of a hospital $h \in H$ satisfy the *law of aggregate demand* if for all $X' \subset X \subset \mathcal{X}_h$, $|C_h(X')| \leq |C_h(X)|$.

Hatfield and Kojima (2009) show that when contracts are substitutes for hospitals and preferences satisfy the law of aggregate demand, the doctor-optimal stable allocation is weakly Pareto optimal for doctors, meaning that there exists no feasible allocation in which all doctors are strictly better off. It follows that there is no sub-problem in which all doctors are strictly better off relative to the doctor-optimal stable allocation.

Theorem 2. For any problem $\pi(T) \in \Pi$, if the preferences of each hospital $h \in H$ satisfy the law of aggregate demand, then there is no $\pi(T') \in \tilde{\Pi}(\pi(T))$ such that for each $d \in D, \overline{X}'_d \succ_d \overline{X}_d$.

When hospitals' preferences satisfy the law of aggregate demand, Theorem 2 implies that withdrawing terms cannot make all doctors better off and, symmetrically, that adding terms cannot make all doctors worse off. However, with $|D| = n$, there are problems in which adding a term makes $n - 1$ doctors worse off.

4 Preference Domains

In this section, we study the conditions on the preference profile that prevent welfare reduction when terms are added. Our main objective is to identify the markets that are vulnerable to this phenomenon.

4.1 Agent-Lexicographic Preferences

We first consider *agent-lexicographic* preferences, in which the ranking is determined primarily by the identity of the agent involved in the contract, with contract terms serving only as secondary criteria. We define the preferences of a doctor d as *hospital-lexicographic* if, for any two acceptable contracts $x, x' \in \mathcal{X}_d$ where $x_h = x'_h = h$, there is no contract $x'' \in \mathcal{X}_d$ with $x''_h \neq h$ such that $x \succ_d x'' \succ_d x'$. For hospitals, the definition differs because hospitals rank sets of contracts rather than individual contracts and are subject to capacity constraints. This requires imposing additional structure on hospital choice functions. For any allocation $X \subseteq \mathcal{X}$, let $D(X) \equiv \{d \in D : X_d \neq \emptyset\}$ denote the set of doctors represented in X. We further require that allocations involving multiple doctors be acceptable to hospitals. The choice function of hospital h is *acceptant* if, for every allocation $X \subseteq \mathcal{X}$, $|C_h(X_h)| = \min\{|D(X_h)|, q_h\}$, that is, hospital h accepts as many contracts as possible, up to its capacity.[17] Since contracts are substitutes for hospitals, it follows that for any $X, Y \subseteq \mathcal{X}_h$ with $Y \subset X$, $C_h(X) \cap Y \subseteq C_h(Y)$, which ensures a consistent ranking over sets of contracts composed of acceptable contracts. Preferences of a hospital h are *doctor-lexicographic* if its choice function is acceptant and, for any allocations $X, Y \subseteq \mathcal{X}$ with $X_h \succ_h Y_h$ and $D(X_h) = D(Y_h)$, there exists no allocation $Z \subseteq \mathcal{X}$ with $D(Z_h) \neq D(X_h)$ such that $X_h \succ_h Z_h \succ_h Y_h$. For convenience, we say that the preferences of an agent

[17] An acceptant choice function induces preferences that satisfy the law of aggregate demand.

$a \in D \cup H$ are agent-lexicographic if, for $a \in D$, the preferences $\succ_a$ are hospital-lexicographic, and for $a \in H$, the preferences $\succ_a$ are doctor-lexicographic. We denote by $\succ_{-a} \equiv (\succ_{a'})_{a' \in (D \cup H) \setminus \{a\}}$ the preference profile of all agents other than a. When all agents have agent-lexicographic preferences, contract terms serve only as secondary criteria, so adding or withdrawing terms does not alter the assignment of doctors to hospitals in the doctor-optimal stable allocation.

Lemma 1. Let $\pi(T) \in \Pi$ be a problem in which each agent $a \in D \cup H$ has agent-lexicographic preferences. Then, for any $\pi(T') \in \tilde{\Pi}(\pi(T))$, it follows for each $h \in H$ that $D(\overline{X}_h) = D(\overline{X}'_h)$.

The result follows immediately, since T'_h is non-empty for every $h \in H$, and we therefore omit the proof. Because each doctor remains assigned to the same hospital and hospitals' choice functions are acceptant, each doctor selects her most-preferred available contract, and doctor-lexicographic preferences ensure that expanding the set of terms does not intensify competition among doctors.[18]

The next result shows that if at least one agent (doctor or hospital) does not have agent-lexicographic preferences, it is always possible to construct a preference profile for the other agents, all satisfying agent-lexicographic preferences, such that expanding the set of available terms reduces the welfare of some doctors.

Theorem 3. If $\succ_a$ are not agent-lexicographic for $a \in D \cup H$, then, there exist $\succ_{-a}$ such that for each $a' \in (D \cup H) \setminus \{a\}$, $\succ_{a'}$ are agent-lexicographic, and a sub-problem $\pi(T') \in \tilde{\Pi}(\pi(T))$, where for some $d \in D$, $\overline{X}'_d \succ_d \overline{X}_d$.

Theorem 3 shows that preventing welfare reductions requires agent-lexicographic preferences on both sides of the market, a condition that is rarely satisfied in practice. For instance, when terms represent salaries, this would require doctors to accept any salary (even negative salaries) to be employed at a hospital, and hospitals to offer any salary (even arbitrarily high salaries) to retain a doctor. Lemma 1 and Theorem 3 together provide the condition, in the maximal-domain sense, that prevents reductions in doctors' welfare when additional terms are introduced for some hospitals.

[18] We formally establish this reasoning in the proof of Proposition 2.

Example 3 illustrates that when agent-lexicographic preferences are imposed on only one side of the market—specifically, hospitals—welfare reductions for doctors may occur following the introduction of contract terms.

Example 3. Consider a problem where $D = \{d_1, d_2\}$, $H = \{h_1, h_2\}$, $\mathcal{T} = \{t, t'\}$ and each hospital has a capacity of one. The vector of terms is given by $T' = (T'_{h_1} = \{t\}, T'_{h_2} = \{t\})$ and let $T_{h_1} = \{t, t'\}$. Contracts are substitutes for hospitals, and only doctor d_1's preferences are not agent-lexicographic:

Doctors:

$$d_1 : (d_1, h_1, t') \succ_{d_1} (d_1, h_2, t) \succ_{d_1} (d_1, h_1, t),$$
$$d_2 : (d_2, h_1, t) \succ_{d_2} (d_2, h_1, t') \succ_{d_2} (d_2, h_2, t).$$

Hospitals:

$$h_1 : (d_1, h_1, t) \succ_{h_1} (d_1, h_1, t') \succ_{h_1} (d_2, h_1, t') \succ_{h_1} (d_2, h_1, t),$$
$$h_2 : (d_1, h_2, t) \succ_{h_2} (d_2, h_2, t).$$

The doctor-optimal stable allocations for $\pi(T')$ and $\pi(T_{h_1}, T'_{-h_1})$ are

$$\overline{X}' = \{(d_2, h_1, t), (d_1, h_2, t)\}, \quad \overline{X} = \{(d_1, h_1, t'), (d_2, h_2, t)\}.$$

The added term is used by doctor d_1, who strictly prefers $\overline{X}$ to $\overline{X}'$, while d_2 strictly prefers $\overline{X}'$ to $\overline{X}$.[19] When hospitals have doctor-lexicographic preferences, new terms can attract doctors who strictly prefer a contract involving a new term—such as doctor d_1 in Example 3—who then compete with and may replace previously employed doctors, such as d_2. Since hospitals rank contracts primarily by doctor identity, if the set of doctors employed by h remains unchanged, no doctor is worse off.[20] The following proposition formalizes this reasoning.

[19] Note that both $\overline{X}'$ and $\overline{X}$ are Pareto efficient. Pakzad-Hurson (2023) examines the connection between efficiency and stability, identifying acyclicity and student-lexicographic preferences as the necessary and sufficient conditions for the existence of a stable and Pareto efficient allocation. In our framework, the latter corresponds to doctor-lexicographic preferences. In Example 3, hospitals' preferences are doctor-lexicographic and *homogeneous*. Even under acyclicity restrictions, expanding the set of terms can reduce doctors' welfare.

[20] We do not impose any restrictions on the ranking of doctors given a contract term. In Example 3, hospital h_1 prefers the contract involving term t to that involving t' for doctor d_1, and the opposite ranking for doctor d_2. Imposing the same ranking of contracts across all doctors does not alter the results.

Proposition 2. For any problem $\pi(T) \in \Pi$ in which each hospital has doctor-lexicographic preferences, consider $h \in H$, $T'_h \subset T_h$ and the sub-problem $\pi(T'_h, T_{-h})$.

(i) If $D(\overline{X}_h) = D(\overline{X}'_h)$, then there is no $d \in D$ such that $\overline{X}'_d \succ_d \overline{X}_d$.

(ii) If there exists a term $t \in T_h \setminus T'_h$ and a contract $x \in \overline{X}_h$ such that $x_t = t$, then there exists a non-empty subset of doctors $D' \subset D$ such that, for each $d' \in D'$, $\overline{X}_{d'} \succ_{d'} \overline{X}'_{d'}$.

Proposition 2 states that when hospitals have doctor-lexicographic preferences, reducing the set of available terms for hospital h does not make any doctor better off if the set of doctors assigned to h remains unchanged. Symmetrically, expanding the set of terms for h does not make any doctor worse off if the set of assigned doctors is unchanged. This is related to Proposition 1(i). Moreover, if at least one term withdrawn at h was previously used, some doctors strictly prefer the original doctor-optimal stable allocation. Symmetrically, if at least one added term at h is used, at least one doctor is strictly better off. This is related to Proposition 1(ii). However, the conditions identified in Proposition 2 do not prevent reductions in doctors' welfare when doctors have hospital-lexicographic preferences. This is illustrated by the following example.

Example 3 (Continued). Consider the following modified preference profiles.

Doctors:

$$d_1 : (d_1, h_1, t) \succ'_{d_1} (d_1, h_1, t'),$$
$$d_2 : (d_2, h_1, t) \succ_{d_2} (d_2, h_1, t') \succ_{d_2} (d_2, h_2, t).$$

Hospitals:

$$h_1 : (d_1, h_1, t') \succ'_{h_1} (d_2, h_1, t') \succ'_{h_1} (d_1, h_1, t) \succ'_{h_1} (d_2, h_1, t),$$
$$h_2 : (d_1, h_2, t) \succ_{h_2} (d_2, h_2, t).$$

Only the preferences of d_1 and h_1 are modified. In particular, hospital h_1 is the only agent whose preferences are not agent-lexicographic. The doctor-optimal stable allocations for $\pi(T')$ and $\pi(T_{h_1}, T'_{-h_1})$ are

$$\overline{X}' = \{(d_1, h_1, t), (d_2, h_2, t)\}, \quad \overline{X} = \{(d_1, h_1, t'), (d_2, h_2, t)\}.$$

The conditions identified in Proposition 2 are satisfied: $D(\overline{X}_{h_1}) = D(\overline{X}'_{h_1})$, and the term $t' \in T_{h_1} \setminus T'_{h_1}$ is used by d_1, while $\overline{X}'_{d_1} \succ_{d_1} \overline{X}_{d_1}$. When doctors have hospital-lexicographic preferences, introducing new terms at hospitals whose own preferences are not doctor-lexicographic may attract doctors who strictly prefer a contract at that hospital to their current allocation. Because the identity of the hospital is their primary criterion, they may be willing to accept less favorable terms, thereby intensifying competition among doctors.

4.2 Common and Polarized Preferences

We now examine the domains of *common* and *polarized preferences.* Preferences over salaries, as illustrated in Example 1, fall within this domain.[21] Doctors prefer higher salaries from a given hospital, while hospitals prefer to hire doctors at lower salaries. We denote $t \gg_D t'$ if, for each doctor $d \in D$ and each hospital $h \in H$ $(d, h, t) \succ_d (d, h, t')$. Doctors' preferences are *common* if, for any distinct $t, t' \in \mathcal{T}$, we have $t \gg_D t'$ or $t' \gg_D t$. For hospitals, as in the definition of doctor-lexicographic preferences, we restrict attention to acceptant choice functions. This ensures that each hospital selects all acceptable contracts up to its capacity. Since contracts are substitutes for hospitals, a hospital's preferences over sets of contracts are then entirely determined by its preferences over individual contracts. We write $t \gg_H t'$ if, for each hospital $h \in H$ and every doctor $d \in D$, $(d, h, t) \succ_h (d, h, t')$. Hospitals' preferences are *common* if hospitals' choice functions are acceptant and, for any distinct $t, t' \in \mathcal{T}$, we have $t \gg_H t'$ or $t' \gg_H t$. Doctors' and hospitals' preferences are *polarized* if doctors' and hospitals' preferences are common and, for any distinct $t, t' \in \mathcal{T}$, $t \gg_D t'$ if and only if $t' \gg_H t$. We say that the term t is *preferred* over t' by doctors if doctors' preferences are common and $t \gg_D t'$.

Example 1 illustrates that even when terms preferred by doctors are introduced, some doctors may be worse off. The next proposition establishes conditions that

[21] In addition to the conditions imposed by polarized preferences, matching with salaries (Kelso and Crawford 1982; Echenique 2012) requires that, if for a given salary a doctor prefers a contract with hospital h to a contract with hospital h', this preference is maintained for any salary. Formally, for any $h, h' \in H$ and $t, t' \in \mathcal{T}$, where t and t' are salaries, $(d, h, t) \succ_d (d, h', t)$ if and only if $(d, h, t') \succ_d (d, h', t')$ for all $d \in D$. A symmetric condition applies to hospitals' common preferences. Schlegel (2015) discusses this restriction and shows that it can be relaxed in some contexts.

prevent such welfare reductions.

Proposition 3. For any problem $\pi(T) \in \Pi$ in which doctors' and hospitals' preferences are polarized, consider $h \in H$, $T'_h \subset T_h$, and the sub-problem $\pi(T'_h, T_{-h})$. If each $t \in T_h \setminus T'_h$ is preferred by doctors to any $t' \in T'_h$, and for each $d' \notin D(\overline{X}'_h)$ one of the following holds,

(i) there exists $t' \in T'_h$ such that $(d', h, t') \succ_{d'} \overline{X}'_{d'}$,

(ii) for each $t \in T_h \setminus T'_h$, $\overline{X}'_{d'} \succ_{d'} (d', h, t)$,

then there is no doctor $d \in D$ such that $\overline{X}'_d \succ_d \overline{X}_d$.

Proposition 3 shows that welfare reductions cannot arise when the introduction of new terms preferred by doctors does not intensify competition among doctors. Condition (ii) captures the case in which doctors not employed by h are uninterested in the newly introduced terms, while condition (i) covers the case where such doctors remain interested in h even under less favorable terms but were not employed there in the original allocation. Since the added terms are preferred by doctors, these conditions are restrictive in practice and are unlikely to be satisfied in many real-world settings. The key insight is that, under preference polarization, any doctor who could be adversely affected by the introduction of new terms can revert to her previous position at the same hospital. The next example illustrates that polarization is crucial for this result, as welfare reductions may arise in its absence.

Example 4. Consider a problem where $D = \{d_1, d_2, d_3\}$, $H = \{h\}$, $\mathcal{T} = \{t, t', t''\}$, and $q_h = 2$. The vectors of terms are given by $T' = (T'_h = \{t', t''\})$ and $T = (T_h = \{t, t', t''\})$. For clarity, and since preferences over sets of contracts are determined by the ranking of individual contracts, we report only individual contracts in the hospital's preferences, including some contracts that are unacceptable to d_3. Contracts are substitutes for the hospital, and both the hospital and the doctors have common preferences:

Doctors:

$$\begin{aligned}
d_1 :& (d_1, h, t) \succ_{d_1} (d_1, h, t') \succ_{d_1} (d_1, h, t''), \\
d_2 :& (d_2, h, t) \succ_{d_2} (d_2, h, t') \succ_{d_2} (d_2, h, t''), \\
d_3 :& (d_3, h, t) \succ_{d_3} (d_3, h, t') \succ_{d_3} \varnothing \succ_{d_3} (d_3, h, t'').
\end{aligned}$$

Hospital:

$$h : (d_1, h, t'') \succ_h (d_3, h, t'') \succ_h (d_2, h, t'') \succ_h (d_1, h, t) \succ_h$$
$$(d_3, h, t) \succ_h (d_2, h, t) \succ_h (d_1, h, t') \succ_h (d_2, h, t') \succ_h (d_3, h, t').$$

The preferences for the terms are as follows: $t \gg_D t' \gg_D t''$ and $t'' \gg_H t \gg_H t'$. The doctor-optimal stable allocations for $\pi(T')$ and $\pi(T)$ are

$$\overline{X}' = \{(d_1, h, t'), (d_2, h, t'), \varnothing\}, \quad \overline{X} = \{(d_1, h, t), (d_2, h, t''), \varnothing\}.$$

Doctor d_1 prefers $\overline{X}_{d_1}$ over $\overline{X}'_{d_1}$, while doctor d_2 prefers $\overline{X}'_{d_2}$ over $\overline{X}_{d_2}$. The added term t is preferred by doctors to the currently available terms (i.e., t' and t''), and condition (i) of Proposition 3 is satisfied since $(d_3, h, t') \succ_{d_3} \overline{X}'_{d_3}$.

Example 1 further illustrates that even when the added term is preferred by doctors and is used, the doctor who uses it may be worse off. To rule out this possibility, we impose a stronger condition ensuring that when terms preferred by doctors are added, if any added term is used, then at least one doctor is strictly better off. This condition requires that the set of doctors employed by h remains unchanged when the set of available terms for h is expanded.

Proposition 4. For any problem $\pi(T) \in \Pi$ in which doctors' and hospitals' preferences are common, consider $h \in H$, $T'_h \subset T_h$, and the sub-problem $\pi(T'_h, T_{-h})$. If

(i) each $t \in T_h \setminus T'_h$ is preferred by doctors to every $t' \in T'_h$,
(ii) $D(\overline{X}_h) = D(\overline{X}'_h)$, and
(iii) there exists a term $t \in T_h \setminus T'_h$ and a contract $x \in \overline{X}_h$ such that $x_t = t$,

then there exists a non-empty subset of doctors $D' \subset D$ such that, for each $d \in D'$, $\overline{X}_d \succ_d \overline{X}'_d$.

Example 4 illustrates that even when the conditions of Proposition 4 are satisfied and doctor d_1 is strictly better off, some other doctors may still be worse off, suggesting that the conditions required to prevent welfare reductions are stringent in practice.[22]

[22] The proof of Proposition 4 is omitted, since at least one doctor previously employed by h is now employed under a preferred contract term, and is therefore strictly better off.

5 Numerical Simulations

Our theoretical results show that doctors' welfare can decrease at the doctor-optimal stable allocation when additional terms are introduced. The only condition that prevents such a decrease is that all agents in the market have agent-lexicographic preferences. Since this condition is rarely satisfied in practice, it is important to quantify, in realistic markets, the shares of doctors who are better off and those who are worse off when new terms are added.

We conduct two sets of simulations in which the parameters influencing doctors' welfare are determined by the market environment—namely, the set of agents and their preferences, the available terms, and hospitals' capacities, consistent with the theoretical model. In each simulation, hospitals are randomly endowed with a subset of terms, the doctor-optimal stable allocation is computed, additional terms are introduced, and the allocation is recomputed. Doctors are classified as worse off if they prefer the initial allocation and better off if they prefer the new allocation.

In the first set of simulations, agents' preferences are drawn randomly, corresponding to markets in which terms do not generate common preferences—such as working hours, medical specialties, or military branches.[23] We fix 10 hospitals with a capacity of 5, vary the number of doctors across 40, 50, and 60, and endow each hospital with an average of 5 terms drawn from a pool of 10. We then introduce up to 1, 2, 3, or 4 additional terms per hospital, running 200 simulations per configuration. Figure 1 reports average shares of better-off and worse-off doctors with 95% confidence intervals.[24]

Both shares increase with the number of additional terms. When doctors exceed available positions—as in the NRMP, where applicants systematically outnumber residency slots—competition is intense and both shares are roughly equal (9–21%),

[23] For hospitals, preferences are defined over individual contracts rather than sets of contracts, which ensures that contracts are substitutes for hospitals. Accordingly, in the COP, each hospital selects, for each doctor, its preferred contract among those proposed by that doctor.

[24] The simulation results are robust to alternative parameter choices. We systematically experimented with different values of these variables and did not observe any qualitative changes.

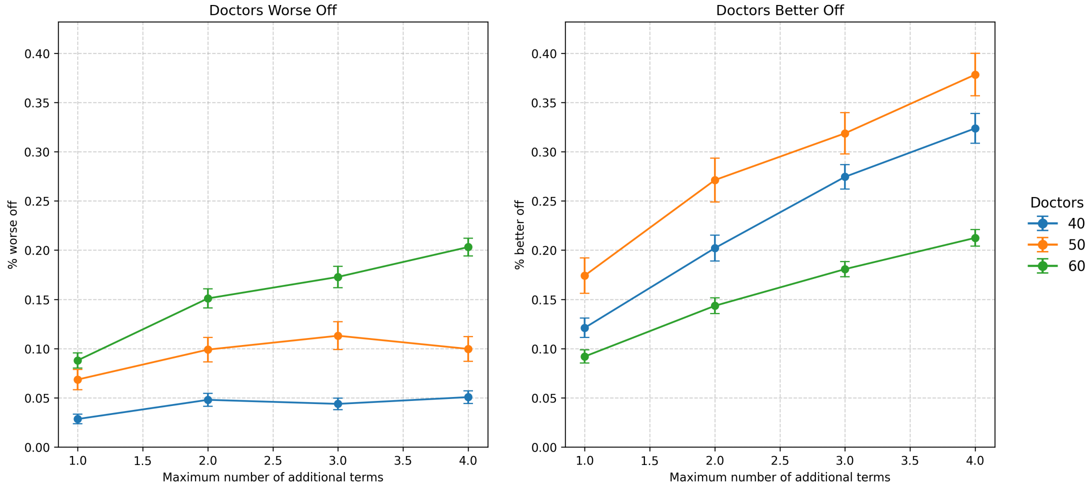

Figure 1: Share of doctors worse off and better off (vertical axis) shown for varying numbers of added terms (horizontal axis), with curves representing different numbers of doctors, after 2,400 simulation runs, with 95% confidence intervals.

as adding terms amplifies this competition.[25] When the number of doctors equals the number of available positions, the share of better-off doctors exceeds that of worse-off doctors, though a non-negligible fraction (7.5–11.5%) still experiences welfare losses. When the number of available positions exceeds the number of doctors, each doctor already secures a favorable contract, leaving limited scope for improvement; welfare losses affect only 3–5% of doctors, and the share of better-off doctors is smaller than in the case of equal numbers of doctors and positions.[26]

In the second set of simulations, we interpret terms as salaries, with each doctor having a minimum acceptable salary at each hospital and each hospital having a maximum salary it is willing to pay for each doctor. Consistent with NRMP data reflecting excess supply, we fix 60 doctors and 10 hospitals with a capacity of 5,

[25] In the 2024 Main Residency Match, 41,503 positions were offered and 38,941 were filled, while the number of active applicants was 44,853. Moreover, certain specialties are particularly competitive relative to the number of available positions, including anesthesiology, medicine-preliminary, neurology, obstetrics-gynecology, pediatrics, and psychiatry. `https://www.nrmp.org/match-data/2024/06/results-and-data-2024-main-residency-match/` (Accessed on 04/14/2026).

[26] We also consider the symmetric case in which the number of doctors is fixed and the number of hospitals varies. The analysis is presented in the Online Appendix and the conclusions remain unchanged.

and 10 salary levels with each hospital initially offering an average of 5. We introduce three categories of additional terms—higher salaries (h) than those initially offered, lower salaries (l), or randomly chosen salaries (r)—running 300 simulations per configuration. Results are reported in Figure 2 with 95% confidence intervals.

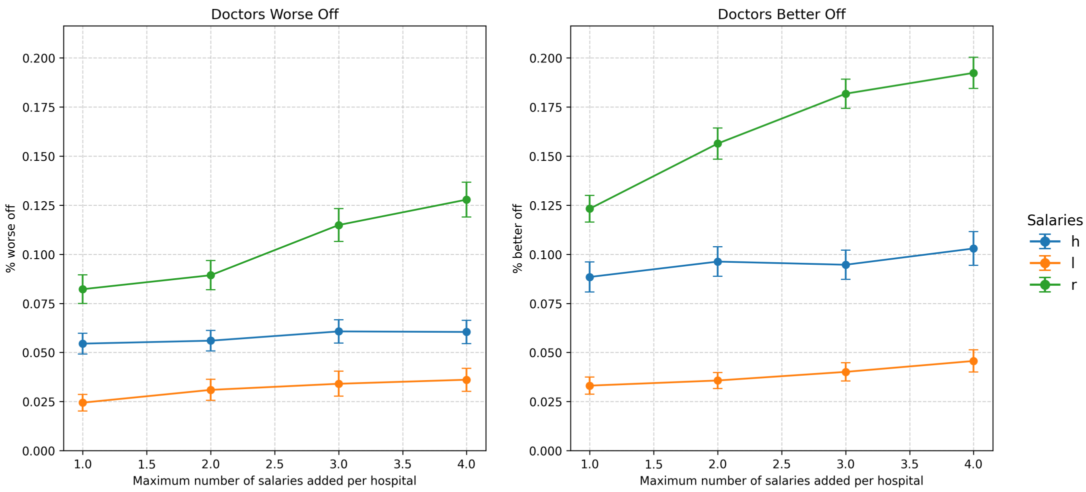

Figure 2: Share of doctors worse off and better off (vertical axis) shown for varying numbers of added terms (horizontal axis), with curves representing the category of added salaries, after 3,600 simulation runs, with 95% confidence intervals.

Both shares increase with the number of additional terms, and the share of better-off doctors exceeds that of worse-off doctors across all scenarios. When lower salaries are introduced, 2.5–3.5% of doctors are worse off and 3–4.5% are better off. Since each doctor has a distinct minimum acceptable salary at each hospital, some accept employment at lower wages, displacing others and generating both gains and losses. When higher salaries are introduced, 5.5–6.5% are worse off and 8.5–10.5% are better off, driven by heterogeneity in doctors' preferences over hospitals, which generates excess demand at some hospitals. When salaries are added at random, both effects occur simultaneously, producing 8–13% worse-off and 12.5–19.5% better-off doctors. These simulations confirm that welfare losses remain non-negligible even under more realistic preferences.

6 Competitive Equilibrium with Transfers

In many markets—centralized or decentralized—salaries for a given position are constrained and therefore impersonal. While such constraints may prevent the existence of a competitive equilibrium, stable allocations are guaranteed to exist as long as contracts are substitutes for hospitals (Kamecke 1998; Hatfield and Kominers 2012). Bulow and Levin (2006) show the negative effects of impersonal salaries on workers. In this section, we show that our results are robust to the choice of solution concept.

We adopt the definitions of Section 2 for the sets of doctors D, hospitals H, terms $\mathcal{T}$, the vector of terms T, and hospitals' capacities q, and consider valuations instead of preferences. For each doctor $d \in D$, let $v_d(x) \in \mathbb{R}$ denote her *valuation* of contract $x \in \mathcal{X}_d \cup \{\varnothing\}$, and let $v_D \equiv (v_d)_{d \in D}$ denote the *profile of doctors' valuations*. For each hospital $h \in H$, let $v_h(X) \in \mathbb{R}$ denote its valuation over sets of contracts $X \in 2^{\mathcal{X}_h} \cup \{\varnothing\}$, and let $v_H \equiv (v_h)_{h \in H}$ denote the *profile of hospitals' valuations*. We write $v_A \equiv (v_a)_{a \in D \cup H}$ for the *agent-valuation* profile, and $v_{-a} \equiv (v_{a'})_{a' \in (D \cup H) \setminus \{a\}}$ for the profile of all agents other than a. We normalize $v_a(\varnothing) = 0$ for all $a \in D \cup H$, and write $v_h(x)$ instead of $v_h(\{x\})$ when $X = \{x\}$.

We assume that each hospital's valuation is *monotone*, that is, for each hospital $h \in H$, $X \subset \mathcal{X}_h$, and $x \in \mathcal{X}_h \setminus X$, $v_h(X \cup \{x\}) \geq v_h(X)$.[27]

Transfers between doctors and hospitals, interpreted as *salaries*, are fully flexible. For a contract $x \in X$ in an allocation X, let $s_x \in \mathbb{R}$ denote the salary paid by hospital x_h to doctor x_d. Since each doctor can be associated with at most one contract, we denote by s the *vector of salaries*. The salary associated with the null contract is normalized to zero, i.e., $s_\varnothing = 0$.

A *problem with salaries* is a tuple (D, H, T, q, v_D, v_H), denoted $\pi_s(T)$ with D, H, q, v_D, and v_H fixed throughout. Let Π_s be the *set of all problems with salaries*.[28] The set of feasible allocations is denoted by $\mathcal{F}(\pi_s(T))$, and the set of sub-problems with salaries derived from $\pi_s(T)$ by $\tilde{\Pi}(\pi_s(T))$.

Given an allocation X and a vector of salaries s, an *allocation with salaries* is

[27] Under monotonicity, the *gross substitutes* condition is equivalent to the *single improvement* property and to the *no complementarities* property (Gul and Stacchetti 1999), which are central to the existence of equilibrium.

[28] See footnote 6 for the formulation.

the pair (X, s). For such a pair, the *utility* of doctor d is given by $u_d(X, s) = v_d(X_d) + s_{X_d}$, and the *profit* of hospital h is given by $p_h(X, s) = v_h(X_h) - \sum_{x \in X_h} s_x$. The set of *available contracts for doctor* d in $\pi_s(T)$ is $\mathcal{X}_d(\pi_s(T)) \equiv \{x \in \mathcal{X}_d : x_t \in T_{x_h}\}$, and the set of *available contract sets for hospital* h is $\mathcal{X}_h(\pi_s(T)) \equiv \{X \subseteq \mathcal{X}_h : |X| \leq q_h, |X_d| \leq 1 \text{ for each } d \in D, \text{ and } x_t \in T_h \text{ for each } x \in X\}$. Given a problem with salaries $\pi_s(T)$ and a vector of salaries s, doctor d's *choice (correspondence)* is $\Phi_d(s) \equiv \arg\max_{X \in \mathcal{X}_d(\pi_s(T)) \cup \{\varnothing\}} u_d(X, s)$, and hospital h's *demand (correspondence)* is $\Phi_h(s) \equiv \arg\max_{X \subseteq \mathcal{X}_h(\pi_s(T)) \cup \{\varnothing\}} p_h(X, s)$.

Definition 4. An allocation with salaries (X, s) is a *competitive equilibrium* if for each doctor d, $X_d \in \Phi_d(s)$, and for each hospital h, $X_h \in \Phi_h(s)$.[29]

Let $\mathcal{E}(\pi_s(T))$ denote the *set of competitive equilibria* for the problem with salaries $\pi_s(T)$. We assume that hospitals' demands satisfy the gross substitutes condition.

Definition 5 (Kelso and Crawford 1982)**.** The demand Φ_h satisfies the *gross substitutes condition* if, for any vectors of salaries s and s', such that $s' \geq s$, and any $X \in \Phi_h(s)$, there exists $X' \in \Phi_h(s')$ such that $\{x \in X : s_x = s'_x\} \subseteq X'$.

When hospitals' demands satisfy the gross substitutes condition, the set of competitive equilibria is non-empty.[30] In particular, there exists a *doctor-optimal competitive equilibrium*, denoted $(\hat{X}, \hat{s})$ for $\pi_s(T)$ and $(\hat{X}', \hat{s}')$ for a sub-problem with salaries $\pi_s(T')$, in which each doctor attains her highest utility across all competitive equilibria. We assume throughout that the induced allocation is generically unique across competitive equilibria, so that all equilibria differ only through transfers. This property holds generically in assignment models (see, Shapley and Shubik 1971). Shapley and Shubik (1971) and Kelso and Crawford (1982) show that this allocation maximizes total *surplus*, defined for an allocation X as $\sum_{d \in D} v_d(X_d) + \sum_{h \in H} v_h(X_h)$. The next example illustrates that adding contract terms can reduce doctors' welfare at the doctor-optimal competitive equilibrium.

[29]Each doctor's choice maximizes her utility over all available contracts, including the null contract, and each hospital's demand maximizes its profit over all available sets of contracts, including the null contract. Therefore, every competitive equilibrium yields weakly positive utility for each doctor and weakly positive profit for each hospital.

[30]Gul and Stacchetti (1999) show that, when hospital demands satisfy the gross substitutes condition, the strict core—as introduced by Kelso and Crawford (1982)—coincides with the set of competitive equilibria.

Example 5. Consider a problem with salaries where $D = \{d_1, d_2\}$, $H = \{h\}$, and $q_h = 1$. Terms represent shifts, with a *night* shift n and a *morning* shift m, giving $\mathcal{T} = \{m, n\}$. The vectors of available terms are $T' = (T'_h) = (\{n\})$ and $T = (T_h) = (\{n, m\})$. Agents' valuations are:

Doctors:

$$d_1 : v_{d_1}((d_1, h, n)) = -20, \quad v_{d_1}((d_1, h, m)) = -5,$$
$$d_2 : v_{d_2}((d_2, h, n)) = -20, \quad v_{d_2}((d_2, h, m)) = 10.$$

Hospital:

$$h : v_h((d_1, h, n)) = 110, \; v_h((d_1, h, m)) = 100, \; v_h((d_2, h, n)) = 80, \; v_h((d_2, h, m)) = 70.$$

The doctor-optimal competitive equilibria for $\pi_s(T')$ and $\pi_s(T)$ are given by

$$\hat{X}' = \{(d_1, h, n), \varnothing\}, \quad \hat{s}' = (50, 0), \quad \hat{X} = \{(d_1, h, m), \varnothing\}, \quad \hat{s} = (20, 0).$$

Under $(\hat{X}', \hat{s}')$, doctors' utilities are $u_{d_1}(\hat{X}', \hat{s}') = 30$ and $u_{d_2}(\hat{X}', \hat{s}') = 0$, whereas once the morning shift is available, they obtain $u_{d_1}(\hat{X}, \hat{s}) = 15$ and $u_{d_2}(\hat{X}, \hat{s}) = 0$. Although d_1 assigns a higher value to the contract with the morning shift, her utility decreases by 15. To understand the source of this reduction, it is useful to compute the surplus generated by each feasible allocation. In $\pi_s(T')$, two contracts are involved in the feasible allocations:

$$v_{d_1}((d_1, h, n)) + v_h((d_1, h, n)) = 90, \quad v_{d_2}((d_2, h, n)) + v_h((d_2, h, n)) = 60.$$

Since d_2 is willing to work whenever her utility is weakly positive and the competitive equilibrium maximizes total surplus, the equilibrium allocation generates a surplus of 90, of which the difference $90 - 60 = 30$ accrues to d_1. When the morning shift becomes available, two additional contracts are involved in the feasible allocations:

$$v_{d_1}((d_1, h, m)) + v_h((d_1, h, m)) = 95, \quad v_{d_2}((d_2, h, m)) + v_h((d_2, h, m)) = 80.$$

By the same argument, the equilibrium allocation includes contract (d_1, h, m). Since d_2 is willing to work as long as her utility is weakly positive, the hospital must obtain a profit of at least 80, reducing d_1's utility to $95 - 80 = 15$. The introduction of additional terms thus intensifies competition by generating more attractive outside options for hospitals.

Despite the similarity in welfare reductions, preventing them under competitive equilibrium requires restrictions on the surplus generated by newly feasible allocations, rather than the preference conditions of Sections 4.1 and 4.2. Indeed, even when all agents' valuations induce agent-lexicographic rankings—as in Example 5, where the hospital assigns strictly higher values to contracts with d_1 than with d_2 regardless of the shift—introducing additional terms may still reduce doctors' welfare.[31] An analogous conclusion holds under polarized preferences. The valuations of contracts involving terms n and m induce a polarized ranking, since both doctors assign higher valuations to contracts with term m than to those with term n, while the hospital assigns higher valuations to contracts with term n. Term m is therefore more valuable for doctors, is introduced, and is used in equilibrium, while the set of doctors assigned to h remains unchanged, so that conditions analogous to those of Proposition 4 are satisfied, although d_1 is worse off.

The welfare reduction stems from the asymmetry in doctors' marginal valuations of the new term. When the morning shift is introduced, $v_{d_1}((d_1, h, m)) - v_{d_1}((d_1, h, n)) = 15$ and $v_{d_2}((d_2, h, m)) - v_{d_2}((d_2, h, n)) = 30$, whereas the hospital's marginal valuation is identical across doctors, with $v_h((d_1, h, m)) - v_h((d_1, h, n)) = v_h((d_2, h, m)) - v_h((d_2, h, n)) = -10$. Since d_2 values the morning shift relatively more than d_1 does, d_2 becomes relatively more attractive when the new term is available, reducing d_1's utility.

We say that an agent-valuation profile v_A *exhibits common marginal term valuations* if, for each $h \in H$, any $d, d' \in D$, and any $t, t' \in \mathcal{T}$,

$$v_d((d, h, t)) - v_d((d, h, t')) = v_{d'}((d', h, t)) - v_{d'}((d', h, t')),$$

and

$$v_h((d, h, t)) - v_h((d, h, t')) = v_h((d', h, t)) - v_h((d', h, t')).$$

[31] Because hospitals' valuations are monotone, each hospital weakly benefits from accepting additional contracts. This allows the induced ordinal ranking to be defined analogously to doctor-lexicographic preferences, together with the acceptance property.

This condition requires that the marginal value of substituting one term for another be identical across all doctors at a given hospital, and symmetrically across all doctors from that hospital's perspective. An agent $a \in D \cup H$ *induces non-common marginal term valuations* for hospital h if this condition is violated because of v_a. Formally, if $a \in D$, there exist $d' \in D$ and $t, t' \in \mathcal{T}$ such that

$$v_a((a,h,t)) - v_a((a,h,t')) \neq v_{d'}((d',h,t)) - v_{d'}((d',h,t')),$$

and if $a \in H$, there exist $d, d' \in D$ and $t, t' \in \mathcal{T}$ such that

$$v_a((d,a,t)) - v_a((d,a,t')) \neq v_a((d',a,t)) - v_a((d',a,t')).$$

The agent-valuation profile of Example 5 violates this condition, as d_1 and d_2 assign different marginal values to the morning shift relative to the night shift. To study the role of this condition in preventing welfare losses, we impose *additive separability* on hospitals' valuations, that is, for each $h \in H$ and each set of contracts $X \in 2^{\mathcal{X}_h} \cup \{\varnothing\}$, $v_h(X) = \sum_{x \in X} v_h(x)$, which restricts how hospitals aggregate individual contract valuations and implies the gross substitutes condition.[32] The next result identifies the maximal domain under which doctors' welfare is not reduced when additional terms are introduced, assuming hospitals' valuations are additively separable.

Theorem 4. Let $\pi_s(T) \in \Pi_s$ be a problem with salaries such that, for each $h \in H$, v_h is additively separable.

(i) If v_A exhibits common marginal term valuations, then for each $\pi_s(T') \in \tilde{\Pi}(\pi_s(T))$, there is no doctor $d \in D$ such that $u_d(\hat{X}', \hat{s}') > u_d(\hat{X}, \hat{s})$.

(ii) Suppose there exists an agent $a \in D \cup H$ that induces non-common marginal term valuations for some hospital. Then there exist an agent-valuation profile v_{-a} that exhibits common marginal term valuations and a sub-problem with salaries $\pi_s(T') \in \tilde{\Pi}(\pi_s(T))$ such that, for some doctor $d \in D$, $u_d(\hat{X}', \hat{s}') > u_d(\hat{X}, \hat{s})$.

[32] This assumption follows Crawford and Knoer (1981) and is relevant for specialized markets such as the labor market for gastroenterologists (Niederle and Roth 2003), or in situations in which resources are relatively homogeneous, such as unskilled labor.

In practice, common marginal term valuations are rarely satisfied, as they require all doctors to assign identical marginal values to any given term at a given hospital. When terms reflect heterogeneous personal circumstances—such as amenities or work schedules—doctors' marginal valuations naturally differ. As a result, most markets are vulnerable to welfare losses when additional terms are introduced. We now show that the results established in Section 3 for stable allocations extend to competitive equilibria.

Proposition 5. For any problem with salaries $\pi_s(T) \in \Pi_s$,

(i) Suppose that $(X, s) \in \mathcal{E}(\pi_s(T))$. For each $\pi_s(T') \in \tilde{\Pi}(\pi_s(T))$, if $X \in \mathcal{F}(\pi_s(T'))$, then $(X, s) \in \mathcal{E}(\pi_s(T'))$.

(ii) For any $\pi_s(T'), \pi_s(T'') \in \tilde{\Pi}(\pi_s(T))$, it holds that $\mathcal{E}(\pi_s(T')) \cap \mathcal{E}(\pi_s(T'')) \subseteq \mathcal{E}(\pi_s(T' \cup T''))$.

Proposition 5 is the analogue of Theorem 1. Part (i) shows that a competitive equilibrium remains such in any sub-problem in which its associated allocation is feasible, while part (ii) establishes the relation between competitive equilibria of sub-problems with salaries and those of the original problem with salaries. Proposition 5(i) implies that if terms not used in the doctor-optimal competitive equilibrium allocation are withdrawn, then the allocation with salaries remains a competitive equilibrium. Moreover, since the competitive equilibrium allocation is unique, the allocation remains unchanged.

Corollary 2. For any problem with salaries $\pi_s(T) \in \Pi_s$, suppose there exists $h \in H$ such that at least one term of h is not used in the allocation with salaries $(\hat{X}, \hat{s})$. Then, there exists $T'_h \subset T_h$ such that $\pi_s(T'_h, T_{-h}) \in \tilde{\Pi}(\pi_s(T))$, with $\hat{X} = \hat{X}'$, and $\hat{s}' \geq \hat{s}$.

Corollary 2 implies that restricting the set of available terms can increase doctors' welfare under competitive equilibrium. As in the case of stable allocations, however, withdrawing terms does not ensure that all doctors become strictly better off, and adding terms does not reduce the utility of every doctor.

Proposition 6. For any problem with salaries $\pi_s(T) \in \Pi_s$, there is no $\pi_s(T') \in \tilde{\Pi}(\pi_s(T))$ such that for each $d \in D$, $u_d(\hat{X}', \hat{s}') > u_d(\hat{X}, \hat{s})$.

7 Conclusion

This paper studies the impact of adding contract terms on doctors' welfare at the doctor-optimal stable allocation. We show that adding terms can reduce doctors' welfare, whereas withdrawing terms that are not used either leads to a Pareto improvement or leaves the allocation unchanged. We identify agent-lexicographic preferences on all agents as a condition that is sufficient—and, in general, necessary—to prevent such welfare reductions. Since this condition is largely theoretical and rarely observed, almost all markets are vulnerable to welfare losses when new terms are introduced. Moreover, because this condition concerns agents' preferences on both sides of the market rather than a directly controllable institutional feature, it offers limited scope for policy intervention. Our simulations confirm this vulnerability, showing that market size and the distribution of preferences are key determinants. When competition among doctors is intense, as in the NRMP, adding terms produces roughly equal shares of doctors who are better or worse off. When terms induce polarized preferences, a non-negligible share of doctors experience welfare losses. These findings highlight the need for careful regulation in the design of terms offered by hospitals. Finally, we show that our results are robust to the choice of solution concept. When competitive equilibrium with flexible transfers is used as the solution concept, the same phenomenon arises, and we identify the corresponding similarities in the results obtained.

Further research may investigate preference domains that limit welfare reductions while preserving realistic preference structures. Identifying such domains could provide guidance for the design of contract terms that improve agents' welfare. Our analysis also motivates empirical work evaluating the welfare consequences of introducing additional terms in markets.

A Additional Examples

In this appendix, we present two additional examples that complement our main analysis. The first example considers salaries, similarly to Example 1, and illustrates that by offering a higher salary, a hospital can retain the same doctors in employment but at a lower salary.

Example 6. Consider a problem where $D = \{d_1, d_2\}, H = \{h_1, h_2\}, \mathcal{T} = \{90, 100, 110\}$, and each hospital has a capacity of one. The vector of terms is given by $T' = (T'_{h_1} = \{90, 100\}, T'_{h_2} = \{100\})$ and let $T_{h_1} = \{90, 100, 110\}$. Contracts are substitutes for hospitals, and preferences are given by:

Doctors:

$$d_1 : (d_1, h_1, 110) \succ_{d_1} (d_1, h_1, 100) \succ_{d_1} (d_1, h_1, 90),$$
$$d_2 : (d_2, h_1, 110) \succ_{d_2} (d_2, h_2, 100).$$

Hospitals:

$$h_1 : (d_1, h_1, 90) \succ_{h_1} (d_2, h_1, 110) \succ_{h_1} (d_1, h_1, 100) \succ_{h_1} (d_1, h_1, 110),$$
$$h_2 : (d_2, h_2, 100).$$

The doctor-optimal stable allocations for $\pi(T')$ and $\pi(T_{h_1}, T'_{-h_1})$ are

$$\overline{X}' = \{(d_1, h_1, 100), (d_2, h_2, 100)\}, \quad \overline{X} = \{(d_1, h_1, 90), (d_2, h_2, 100)\}.$$

Although hospital h_1 offers a higher salary, d_1's salary decreases to 90. In contrast to Example 1, the doctors remain employed at the same hospitals. The source of the welfare reduction is similar: by offering a higher salary, competition arises between d_1 and d_2, ultimately forcing d_1 to accept a lower salary to secure a position at h_1.

The second example considers a setting in which hospitals must, in addition to offering the same terms to all doctors, employ them under identical conditions. This type of constraint is typically imposed in the context of working hours or salaries.

Example 7. Consider a problem where $D = \{d_1, d_2, d_3\}$, $H = \{h\}$, $\mathcal{T} = \{t, t'\}$, and $q_h = 2$. The vectors of terms are given by $T' = (T'_h = \{t'\})$ and $T = (T_h = \{t, t'\})$. Contracts are substitutes for the hospital, and preferences are given by:

Doctors:

$$d_1 : (d_1, h, t') \succ_{d_1} (d_1, h, t), \quad d_2 : (d_2, h, t') \succ_{d_2} (d_2, h, t), \quad d_3 : (d_3, h, t) \succ_{d_3} (d_3, h, t').$$

Hospital:

$$h : \{(d_1, h, t), (d_2, h, t)\} \succ_h \{(d_1, h, t)\} \succ_h \{(d_2, h, t)\} \succ_h \{(d_3, h, t)\} \succ_h \\ \{(d_1, h, t'), (d_2, h, t')\} \succ_h \{(d_1, h, t')\} \succ_h \{(d_2, h, t')\} \succ_h \{(d_3, h, t')\}.$$

The doctor-optimal stable allocations for $\pi(T')$ and $\pi(T)$ are

$$\overline{X}' = \{(d_1, h, t'), (d_2, h, t'), \varnothing\}, \quad \overline{X} = \{(d_1, h, t), (d_2, h, t), \varnothing\}.$$

Although doctors must be employed under the same terms, both d_1 and d_2 are worse off. Such a restriction, intended to reduce inequalities among workers, does not prevent a decrease in welfare.

B Proofs

B.1 Proof of Proposition 1

Proof. For the first part of (i) in Proposition 1 and (ii), we use Example 1. We use strict preferences and stability for the second part of (i). There exists some $d' \in D'$ such that $\overline{X}'_{d'} \succ_{d'} \overline{X}_{d'}$. Suppose, for contradiction, that d' is employed by h in $\overline{X}'$, that is, $\overline{X}'_{d'} = \{(d', h, t)\}$, and that $\overline{X}'_h \succ_h \overline{X}_h$ or $\overline{X}'_h = \overline{X}_h$. By strict preferences, $\overline{X}'_h \neq \overline{X}_h$ because $\overline{X}'_{d'} \succ_{d'} \overline{X}_{d'}$. Using the definition of stability, $\overline{X}'_h$ cannot be preferred by h to $\overline{X}_h$. If both d' and h prefer $\overline{X}'$ to $\overline{X}$, then $\overline{X}$ is not stable. Since contracts are substitutes for h, $\overline{X}'_{d'} \in C_h(\overline{X}' \cup \overline{X})$, it follows that $\overline{X}'_{d'}$ participates in an allocation that blocks $\overline{X}$. Therefore, $\overline{X}_h \succ_h \overline{X}'_h$. ■

B.2 Proof of Theorem 1

Proof. (i) Consider an allocation $X \in S(\pi(T))$ such that $X \in \mathcal{F}(\pi(T'))$. Suppose, for contradiction, that $X \notin S(\pi(T'))$. Let X' denote an allocation that blocks X in $\pi(T')$ with $X \neq X'$ and $X' \in \mathcal{F}(\pi(T'))$, by definition of the choice functions. By construction of T', we know that $X' \in \mathcal{F}(\pi(T))$ and that contracts are substitutes for hospitals. Since X' blocks X in $\pi(T')$, X' also blocks X in $\pi(T)$ because there is no x such that $x \notin C_h(X')$ and $x \in C_h(X' \cup X)$ for some h. Then, $X \notin S(\pi(T))$, leading to a contradiction.

Before proving (ii), we introduce Lemma 2. If an allocation ceases to be stable after the introduction of additional terms, then some of these newly introduced terms must be used in an allocation that blocks the original stable allocation.

Lemma 2. Consider $\pi(T) \in \Pi$ and $\pi(T') \in \tilde{\Pi}(\pi(T))$, and suppose there exists $X' \in S(\pi(T'))$ and $X' \notin S(\pi(T))$. Then there exist $h \in H$, $t \in T_h \setminus T'_h$, and $x \in X$ with $x_h = h$ and $x_t = t$, such that $x \succ_{x_d} X'_{x_d}$ and $X_h \succ_h X'_h$.

Proof. Consider the definition of stability, the construction of $\pi(T)$ and $\pi(T')$, and that contracts are substitutes for hospitals. Let X be an allocation that blocks X' in $\pi(T)$. Suppose, for contradiction, that for each contract $x \in X$, $x_t \in T'_{x_h}$. Since contracts are substitutes for hospitals, $X = C_h(X' \cup X) \subseteq C_D(X' \cup X)$ for some $h \in H$. But then X' would be blocked in $\pi(T')$, contradicting its stability. Therefore, there must exist at least one contract $x \in X$ using a term $t \in T_{x_h} \setminus T'_{x_h}$. ■

(ii) Suppose, for contradiction, that there exists $X' \in S(\pi(T')) \cap S(\pi(T''))$ and $X' \notin S(\pi(T' \cup T''))$. By Lemma 2, there exists $X \in \mathcal{F}(\pi(T' \cup T''))$ with $X \notin (\mathcal{F}(\pi(T')) \cup \mathcal{F}(\pi(T'')))$ that blocks X', since otherwise, X' would not be stable in $\pi(T')$ and $\pi(T'')$. We show that X cannot involve terms from both $T' \setminus T''$ and $T'' \setminus T'$. Suppose there exist $x', x'' \in X$ such that $x'_t \in T'_{x'_h} \setminus T''_{x'_h}$ and $x''_t \in T''_{x''_h} \setminus T'_{x''_h}$. Since contracts are substitutes for hospitals, if $x' \in C_{x'_h}(X'_{x'_h} \cup \{x', x''\})$, then substitutability implies $x' \in C_{x'_h}(X'_{x'_h} \cup \{x'\})$, contradicting the stability of X' in $\pi(T')$. Similarly, if $x'' \in C_{x''_h}(X'_{x''_h} \cup \{x'', x'\})$, then $x'' \in C_{x''_h}(X'_{x''_h} \cup \{x''\})$, contradicting the stability of X' in $\pi(T'')$. Hence, $X' \in S(\pi(T' \cup T''))$. ■

B.3 Proof of Theorem 2

Proof. Suppose that every hospital choice function satisfies the law of aggregate demand for the problem $\pi(T)$, and that $\overline{X}'_d \succ_d \overline{X}_d$ for every $d \in D$, where $\pi(T') \in \tilde{\Pi}(\pi(T))$. By Corollary 1 of Hatfield and Kojima (2009), there exists no allocation $X \in \mathcal{F}(\pi(T))$ such that $X \succ_h \varnothing$ for every $h \in H$ and for every $d \in D$, $X_d \succ_d \overline{X}_d$. The contradiction is immediate, since by construction of T', it follows that $\overline{X}' \in \mathcal{F}(\pi(T))$ and $\overline{X}' \succ_h \varnothing$ for every $h \in H$ as $\overline{X}' \in S(\pi(T'))$. ■

B.4 Proof of Theorem 3

Proof. By the definition of agent-lexicographic preferences, if a hospital h does not have doctor-lexicographic preferences, then $|T_h| \geq 2$ and $|D| \geq 2$. We may restrict attention to the case with exactly two doctors, d_1 and d_2, and a single hospital h with $q_h = 1$, assuming that all other doctors strictly prefer the null contract $\varnothing$ to any contract. Similarly, if a doctor d does not have hospital-lexicographic preferences, there exists at least one hospital h with $|T_h| \geq 2$ and $|H| \geq 2$. We may restrict attention to the case with exactly two doctors, d_1 and d_2, and two hospitals, h_1 and h_2, with $q_{h_1} = q_{h_2} = 1$, assuming that all other doctors strictly prefer the null contract $\varnothing$ to any contract.

(i) Suppose $\succ_h$ are not doctor-lexicographic, and there exist some $x, x', y, y' \in \mathcal{X}$, with $x = (d_1, h, t), x' = (d_1, h, t'), y = (d_2, h, t)$ and $y' = (d_2, h, t')$. We consider four cases:

Case (i)-1: $\succ_h$: $\{x\} \succ_h \{y\} \succ_h \{x'\} \succ_h \{y'\}$. Consider $\succ_{d_1}$: $x' \succ_{d_1} x \succ_{d_1} \varnothing$ and $\succ_{d_2}$: $y \succ_{d_2} y' \succ_{d_2} \varnothing$. Let $T_h = \{t, t'\}$ and $T'_h = \{t'\}$. Since $C_h(\{x', y'\}) = \{x'\}$, it follows that $\overline{X}'_{d_1} = \{x'\}$. In contrast, $C_h(\{x', y\}) = \{y\}$ and $C_h(\{x', y, x\}) = \{x\}$, we have that $\overline{X}_{d_1} = \{x\}$. Therefore, $\overline{X}'_{d_1} \succ_{d_1} \overline{X}_{d_1}$.

Case (i)-2: $\succ_h$: $\{x'\} \succ_h \{y\} \succ_h \{x\} \succ_h \{y'\}$. Consider $\succ_{d_1}$: $x \succ_{d_1} x' \succ_{d_1} \varnothing$ and $\succ_{d_2}$: $y \succ_{d_2} y' \succ_{d_2} \varnothing$. Let $T_h = \{t, t'\}$ and $T'_h = \{t\}$. Since $C_h(\{x, y\}) = \{y\}$, it follows that $\overline{X}'_{d_2} = \{y\}$. In contrast, $C_h(\{x, y\}) = \{y\}$ and $C_h(\{x', y, x\}) = \{x'\}$, we have that $\overline{X}_{d_2} = \{\varnothing\}$. Therefore, $\overline{X}'_{d_2} \succ_{d_2} \overline{X}_{d_2}$.

Case (i)-3: $\succ_h$: $\{x\} \succ_h \{y\} \succ_h \{y'\} \succ_h \{x'\}$. Consider $\succ_{d_1}$: $x \succ_{d_1} x' \succ_{d_1} \varnothing$ and $\succ_{d_2}$: $y' \succ_{d_2} y \succ_{d_2} \varnothing$. Let $T_h = \{t, t'\}$ and $T'_h = \{t'\}$. Since $C_h(\{x', y'\}) = \{y'\}$, it follows that $\overline{X}'_{d_2} = \{y'\}$. In contrast, $C_h(\{x, y'\}) = \{x\}$ and $C_h(\{x, y, y'\}) = \{x\}$, we have that $\overline{X}_{d_2} = \{\varnothing\}$. Therefore, $\overline{X}'_{d_2} \succ_{d_2} \overline{X}_{d_2}$.

Case (i)-4: $\succ_h$: $\{x\} \succ_h \{y'\} \succ_h \{y\} \succ_h \{x'\}$. Consider $\succ_{d_1}$: $x \succ_{d_1} x' \succ_{d_1} \varnothing$ and $\succ_{d_2}$: $y' \succ_{d_2} y \succ_{d_2} \varnothing$. Let $T_h = \{t, t'\}$ and $T'_h = \{t'\}$. Since $C_h(\{x', y'\}) = \{y'\}$, it follows that $\overline{X}'_{d_2} = \{y'\}$. In contrast, $C_h(\{x, y'\}) = \{x\}$ and $C_h(\{x, y, y'\}) = \{x\}$, we have that $\overline{X}_{d_2} = \{\varnothing\}$. Therefore, $\overline{X}'_{d_2} \succ_{d_2} \overline{X}_{d_2}$.

(ii) Suppose $\succ_{d_1}$ are not hospital-lexicographic, this implies that there exist $h_1, h_2 \in H$, with $h_1 \neq h_2$. Further, suppose that there exist some $x_1, x'_1, x_2, y_1, y'_1, y_2 \in \mathcal{X}$, with $x_1 = (d_1, h_1, t), x'_1 = (d_1, h_1, t'), x_2 = (d_1, h_2, t), y_1 = (d_2, h_1, t), y'_1 =$

(d_2, h_1, t') and $y_2 = (d_2, h_2, t)$. We consider two cases:

Case (ii)-1: $\succ_{d_1}$: $x_1 \succ_{d_1} x_2 \succ_{d_1} x_1' \succ_{d_1} \varnothing$. Consider $\succ_{d_2}$: $y_1' \succ_{d_2} y_1 \succ_{d_2} y_2 \succ_{d_2} \varnothing$, $\succ_{h_1}$: $\{x_1\} \succ_{h_1} \{x_1'\} \succ_{h_1} \{y_1\} \succ_{h_1} \{y_1'\}$, and h_2, that finds all contracts involving it acceptable. Let $T_{h_1} = \{t, t'\}$ and $T_{h_1}' = \{t'\}$. Since $C_{h_1}(\{y_1'\}) = \{y_1'\}$, it follows that $\overline{X}_{d_2}' = \{y_1'\}$. In contrast, $C_{h_1}(\{x_1, y_1'\}) = \{x_1\}$ and $C_{h_1}(\{x_1, y_1, y_1'\}) = \{x_1\}$, we have that $\overline{X}_{d_2} = \{y_2\}$. Therefore, $\overline{X}_{d_2}' \succ_{d_2} \overline{X}_{d_2}$.

Case (ii)-2: $\succ_{d_1}$: $x_1' \succ_{d_1} x_2 \succ_{d_1} x_1 \succ_{d_1} \varnothing$. Consider $\succ_{d_2}$: $y_1' \succ_{d_2} y_1 \succ_{d_2} y_2 \succ_{d_2} \varnothing$, $\succ_{h_1}$: $\{x_1\} \succ_{h_1} \{x_1'\} \succ_{h_1} \{y_1\} \succ_{h_1} \{y_1'\}$, and h_2, that finds all contracts involving it acceptable. Let $T_{h_1} = \{t, t'\}$ and $T_{h_1}' = \{t\}$. Since $C_{h_1}(\{y_1\}) = \{y_1\}$, it follows that $\overline{X}_{d_2}' = \{y_1\}$. In contrast, $C_{h_1}(\{x_1', y_1'\}) = \{x_1'\}$ and $C_{h_1}(\{x_1', y_1, y_1'\}) = \{x_1'\}$, we have that $\overline{X}_{d_2} = \{y_2\}$. Therefore, $\overline{X}_{d_2}' \succ_{d_2} \overline{X}_{d_2}$.

By relabeling doctors, hospitals and terms, all symmetric cases follow by the same argument and are omitted. This concludes the proof. ■

B.5 Proof of Proposition 2

We use the construction of the COP to prove Proposition 2. For each $a \in D \cup H$, let $R_a(X) \equiv X - C_a(X)$ denote the set of contracts rejected by a from X.

Cumulative Offer Process.

Step 0. Consider problem $\pi(T)$. For each $d \in D$, let $X_d(0) \equiv \{x \in \mathcal{X}_d : x_t \in T_{x_h}\} \cup \{\varnothing\}$ be the set of contracts available for d.

Step $k \geq 1$. Each $d \in D$ chooses a contract from $X_d(k-1)$ using $C_d(\cdot)$. Let $X_d(k) \equiv C_d(X_d(k-1))$ be the chosen contract. Let $X(k) \equiv \bigcup_{d \in D} C_d(X_d(k-1))$ be the set of all chosen contracts. Each doctor d proposes to hospital $X_d(k)_h$ contract $X_d(k)$. For each $h \in H$, let $X_h(k) \equiv \{X_d(k) \in X(k) : X_d(k)_h = h\}$ be the set of contracts proposed to h. Each hospital h chooses contracts from $X_h(k)$ according to $C_h(\cdot)$ and rejects the remaining contracts, denoted $R_h(k)$. If $R_h(k) = \emptyset$ for all $h \in H$, the procedure terminates. Otherwise, for each $d \in D$, with $X_d(k) \in R(k)$, we let $X_d(k) \equiv X_d(k-1) \setminus \{X_d(k)\}$ and for each $d \in D$ such that $X_d(k) \notin R(k)$, we let $X_d(k) \equiv X_d(k-1)$, and proceed to Step *k+1*.

The algorithm continues until Step k^*, where $R_h(k^*) = \emptyset$ for all $h \in H$. The final allocation is the doctor-optimal stable allocation $\overline{X}$ for the problem $\pi(T)$. Since there are finitely many doctors, hospitals, and possible contracts for each doctor-hospital pair, the COP terminates in a finite number of steps.

Proof. Consider T and T' such that for each $h' \in H \setminus \{h\}, T_{h'} = T'_{h'}$ and $T'_h \subset T_h$, with all hospitals having doctor-lexicographic preferences.

(i) Suppose, for contradiction, that $D(\overline{X}_h) = D(\overline{X}'_h)$ and there exists $d \in D$ such that $\overline{X}'_d \succ_d \overline{X}_d$. We have two cases:

Case (i)-1: d is employed by h in both $\overline{X}$ and $\overline{X}'$. Since h's choice function is acceptant, it follows that during the COP, $\overline{X}'_d$ is rejected by h. Because contracts are substitutes for h, there exists a contract x whose proposal to h causes $\overline{X}'_d$ to be rejected. Since only the ranking over contracts is relevant for this comparison, it then follows that $\{x\} \succ_h \overline{X}'_d$. By construction of the COP, doctor d does not propose any other contract until $\overline{X}'_d$ is rejected, so $x_d \neq d$. Since $D(\overline{X}_h) = D(\overline{X}'_h)$, at a later step of the COP, $\overline{X}_d$ is chosen by h. It follows that h does not prefer x to $\overline{X}_d$, and thus $\overline{X}_d \succ_h \{x\}$. Since h's preferences are strict we have $\overline{X}_d \succ_h \{x\} \succ_h \overline{X}'_d$, which contradicts that h's preferences are doctor-lexicographic.

Case (i)-2: d is not employed by h in $\overline{X}$ and $\overline{X}'$. Let h' denote the hospital employing d in $\overline{X}'$. Since $\overline{X}'_d \succ_d \overline{X}_d$, there exists an allocation $X \neq C_{h'}(\overline{X}')$ such that $X = C_{h'}(\overline{X}' \cup X) \subseteq C_D(\overline{X}' \cup X)$, i.e., X blocks $\overline{X}'$. However, $T_{h'} = T'_{h'}$, so X is also feasible in $\pi(T')$, that is, $X \in \mathcal{F}(\pi(T'))$. This implies that $\overline{X}'$ is not stable in $\pi(T')$, which is a contradiction.

(ii) Suppose for contradiction that $t \in T_h \setminus T'_h$ is used and there is no doctor d such that $\overline{X}_d \succ_d \overline{X}'_d$. Consider $x \in \overline{X}$ with $x_h = h, x_t = t$ and $x_d = d$. Since $x_h = h$, d is hired by h under $\overline{X}$, that is, $d \in D(\overline{X}_h)$, we distinguish two cases:

Case (ii)-1: d is employed by h under $\overline{X}'$. Since h has doctor-lexicographic preferences and $x_h = h$, at the doctor-optimal stable allocation d chooses her most preferred contract among those whose terms are available at h, that is, the contract $x' \in \mathcal{X}_d \cap \mathcal{X}_h$ and $x'_t \in T_h$. By construction of T_h, the term used in $\overline{X}'_d$ is available at h. Moreover, because the choice function of h is acceptant, this contract is not rejected by h, and thus $x' = x$. Since preferences are strict and the term used differs, it follows that $\overline{X}_d \neq \overline{X}'_d$ and $\overline{X}_d \succeq_d \overline{X}'_d$, yielding $\overline{X}_d \succ_d \overline{X}'_d$.

Case (ii)-2: d is not employed by h under $\overline{X}'$. Let h' denote the hospital employing d in $\overline{X}'$. Since additional terms are introduced only at h, we have $T'_{h'} = T_{h'}$, and $\overline{X}'_d$ uses a term available at h'. Because contracts are substitutes for h' and the choice function of h' is acceptant, if d proposes $\overline{X}'_d$ to h', in the COP, it will not be rejected. It follows that $\overline{X}_d \neq \overline{X}'_d$ and $\overline{X}_d \succeq_d \overline{X}'_d$, yielding $\overline{X}_d \succ_d \overline{X}'_d$. These two contradictions conclude the proof. ■

B.6 Proof of Proposition 3

Proof. Let $\pi(T) \in \Pi$ be a problem such that doctors' and hospitals' preferences are polarized. Consider a sub-problem $\pi(T'_h, T_{-h}) \in \tilde{\Pi}(\pi(T))$ such that each $t \in T_h \setminus T'_h$ is preferred by doctors to any $t' \in T'_h$, and suppose that for each $d' \notin D(\overline{X}'_h)$, either there exists $t' \in T'_h$ such that $(d', h, t') \succ_{d'} \overline{X}'_{d'}$, or $\overline{X}'_{d'} \succ_{d'} (d', h, t)$ for all $t \in T_h \setminus T'_h$. We show that $\overline{X}'$ is stable in $\pi(T)$.
Consider any hospital $h' \in H \setminus \{h\}$. A blocking set involving h' would require the existence of a set of contracts X such that $X \in C_{h'}(\overline{X}'_{h'} \cup X)$ and $X = C_d(\overline{X}'_d \cup X)$ with $\overline{X}'_d \neq X$ for some $d \in D$. Since $T_{h'} = T'_{h'}$ and $\overline{X}' \in S(\pi(T'_h, T_{-h}))$, it follows that no such X exists. We now consider hospital h. Any blocking set involving h must include at least one contract (d, h, t) with $t \in T_h \setminus T'_h$. For any such contract, polarization implies that each $d \in D(\overline{X}'_h)$, $(d, h, t) \succ_d \overline{X}'_d$ and $\overline{X}'_d \succ_h \{(d, h, t)\}$. Hence, such contracts cannot be jointly selected by both C_h and C_d when added to $\overline{X}'$. Finally, consider any $d' \notin D(\overline{X}'_h)$. If there exists $t' \in T'_h$ such that $(d', h, t') \succ_{d'} \overline{X}'_{d'}$, then for each $d \in D(\overline{X}'_h)$, $\overline{X}'_d \succ_h \{(d', h, t')\}$, and by polarization it follows that $\overline{X}'_d \succ_h \{(d', h, t)\}$ for each $t \in T_h \setminus T'_h$. Otherwise, if for each $t \in T_h \setminus T'_h$, $\overline{X}'_{d'} \succ_{d'} \{(d', h, t)\}$, then d' does not select any contract involving h. In both cases, no contract involving d' and h can belong to any set X such that $X \in C_h(\overline{X}'_h \cup X)$ and $X = C_{d'}(\overline{X}'_{d'} \cup X)$. Therefore, $\overline{X}' \in S(\pi(T))$, and thus for each $d \in D$, $\overline{X} \succeq_d \overline{X}'$. ■

B.7 Proof of Theorem 4

Proof. Consider a problem with salaries (D, H, T, q, v_D, v_H) such that each hospital $h \in H$ has an additively separable valuation function.
(i) Suppose that v_A exhibits common marginal term valuations. Construct an equivalent unit-capacity problem with salaries by replacing each hospital $h \in H$ with q_h copies $h^* \equiv \{h_1, \ldots, h_{q_h}\}$, each inheriting h's valuation function, that is, $v_\eta(x) = v_h(x)$, with $q_\eta = 1$, $T_\eta = T_h$, and $T'_\eta = T'_h$ for each $\eta \in h^*$. Let

$\mathrm{H}^* \equiv \bigcup_{h \in H} h^*$. Doctors evaluate contracts with any copy η exactly as with h. The original problem with salaries and the transformed unit-capacity problem with salaries $(D, \mathrm{H}^*, T, q^*, v_D, v_{\mathrm{H}^*})$ yield the same profits and salaries.

Let $\pi_s(T') \in \tilde{\Pi}(\pi_s(T))$. We use the classical dual characterization together with surplus maximization of competitive equilibrium allocations (Shapley and Shubik 1971). In particular, $\hat{X}'$ maximizes surplus in $\pi_s(T')$. Under common marginal term valuations, any $t \in T_\eta \setminus T'_\eta$ changes the surplus of pair (d, η) by the same amount for all $d \in D$, so that the surplus-maximizing assignment of doctors to hospital copies is identical in $\pi_s(T)$ and $\pi_s(T')$. Hence, for each $\eta \in \mathrm{H}^*$, $D(\hat{X}'_\eta) = D(\hat{X}_\eta)$. Let d_η be the doctor assigned to η in $\hat{X}'$, that is, $D(\hat{X}'_\eta) = \{d_\eta\}$ and the surplus-maximizing term in $\pi_s(T)$ is $t^*_\eta \in \arg\max_{t \in T_\eta}\{v_\eta((d_\eta, \eta, t)) + v_{d_\eta}((d_\eta, \eta, t))\}$. For each doctor $d \in D$, define $\overline{u_d} \equiv u_d(\hat{X}', \hat{s}')$, and for each $\eta \in \mathrm{H}^*$, let $\overline{p_\eta} \equiv \max_{t' \in T'_\eta} v_\eta((d_\eta, \eta, t')) + v_{d_\eta}((d_\eta, \eta, t')) - \overline{u_{d_\eta}}\}$. Let $\overline{u_D} \equiv (\overline{u_d})_{d \in D}$ and $\overline{p_{\mathrm{H}^*}} \equiv (\overline{p_\eta})_{\eta \in \mathrm{H}^*}$. We show that $(\overline{u_D}, \overline{p_{\mathrm{H}^*}})$ satisfies the dual feasibility constraints for $\pi_s(T)$, that is, for all $d \in D$, $\eta \in \mathrm{H}^*$ and $t \in T_\eta$,

$$\overline{u_d} + \overline{p_\eta} \geq v_d(d, \eta, t) + v_h(d, \eta, t). \tag{1}$$

By common marginal term valuations, for all $d \in D$ and $t \in T_\eta$,

$$\begin{aligned} &v_\eta((d, \eta, t)) + v_d((d, \eta, t)) - v_\eta((d_\eta, \eta, t)) - v_{d_\eta}((d_\eta, \eta, t)) = \\ &\qquad v_\eta((d, \eta, t^*_\eta)) + v_d((d, \eta, t^*_\eta)) - v_\eta((d_\eta, \eta, t^*_\eta)) - v_{d_\eta}((d_\eta, \eta, t^*_\eta)). \end{aligned}$$

By definition of t^*_η, the right-hand side is non-positive, so

$$v_\eta((d, \eta, t)) + v_d((d, \eta, t)) \leq v_\eta((d, \eta, t^*_\eta)) + v_d((d, \eta, t^*_\eta)),$$

for all $d \in D$. It thus suffices to verify (1) at $t = t^*_\eta$. If $t^*_\eta \in T'_\eta$, this follows from the dual characterization of $(\hat{X}', \hat{s}')$ in $\pi_s(T')$ combined with $\overline{p_\eta} \geq p_\eta(\hat{X}', \hat{s}')$. If $t^*_\eta \in T_\eta \setminus T'_\eta$, fix any $t_0 \in T'_\eta$. By commonality,

$$\begin{aligned} &v_\eta((d, \eta, t^*_\eta)) + v_d((d, \eta, t^*_\eta)) - v_\eta((d, \eta, t_0)) - v_d((d, \eta, t_0)) = \\ &\qquad v_\eta((d_\eta, \eta, t^*_\eta)) + v_{d_\eta}((d_\eta, \eta, t^*_\eta)) - v_\eta((d_\eta, \eta, t_0)) - v_{d_\eta}((d_\eta, \eta, t_0)). \end{aligned}$$

Since $(\hat{X}', \hat{s}')$ satisfies the dual constraints at $t_0 \in T'_\eta$, inequality (1) at t^*_η follows from the definition of $\overline{p_\eta}$. Finally, since $\overline{u_d} \geq 0$ and $\overline{p_\eta} \geq 0$, the dual characterization implies $u_d(\hat{X}, \hat{s}) \geq \overline{u_d} = u_d(\hat{X}', \hat{s}')$ for each $d \in D$.

(ii) Suppose there exists an agent $a \in D \cup H$ that induces non-common marginal term valuations for some hospital h. We consider the simplest environment exhibiting the phenomenon: two doctors d_1, d_2, one hospital h with capacity $q_h = 1$, and two terms t, t'. All other doctors and hospitals assign strictly negative valuations to every contract and are thus assigned with the null contract $\varnothing$. Let $x = (d_1, h, t), x' = (d_1, h, t'), y = (d_2, h, t)$ and $y' = (d_2, h, t')$.

Case 1: $a = d_1$. Then $v_{d_1}(x) - v_{d_1}(x') \neq v_{d_2}(y) - v_{d_2}(y')$. Without loss of generality, suppose $v_{d_1}(x) - v_{d_1}(x') > v_{d_2}(y) - v_{d_2}(y')$. Let $T'_h = \{t'\}$ and $T_h = \{t, t'\}$ and consider a valuation profile $\tilde{v}_A$ with $\tilde{v}_{d_1} = v_{d_1}$ and $\tilde{v}_{d_2}(y') + \tilde{v}_h(y') > \tilde{v}_{d_2}(y) + \tilde{v}_h(y) > 0$ together with

$$\tilde{v}_{d_2}(y') + \tilde{v}_h(y') \;>\; \tilde{v}_{d_1}(x) + \tilde{v}_h(x) \;>\; \tilde{v}_{d_1}(x') + \tilde{v}_h(x') > 0. \tag{2}$$

Since $(D \cup H) \setminus \{d_1\} = \{d_2, h\}$, the profile v_{-d_1} naturally exhibits common marginal term valuations. Such a profile exists because $v_{d_1}(x) - v_{d_1}(x') > v_{d_2}(y) - v_{d_2}(y')$, which allows the surpluses in (2) to be ordered as required. The doctor-optimal competitive equilibria are $(\hat{X}', \hat{s}')$ and $(\hat{X}, \hat{s})$ with $\hat{X}' = \hat{X} = \{y', \varnothing\}$ and corresponding vectors of salaries

$$\hat{s}' = \Big(0, \tilde{v}_{d_2}(y') + \tilde{v}_h(y') - (\tilde{v}_{d_1}(x') + \tilde{v}_h(x'))\Big), \quad \hat{s} = \Big(0, \tilde{v}_{d_2}(y') + \tilde{v}_h(y') - (\tilde{v}_{d_1}(x) + \tilde{v}_h(x))\Big).$$

The strict inequality in (2) implies $u_{d_2}(\hat{X}', \hat{s}') > u_{d_2}(\hat{X}, \hat{s})$.

Case 2: $a = h$. Then $v_h(x) - v_h(x') \neq v_h(y) - v_h(y')$. Without loss of generality, suppose $v_h(x) - v_h(x') > v_h(y) - v_h(y')$. Let $T'_h = \{t'\}$ and $T_h = \{t, t'\}$ and consider a valuation profile $\tilde{v}_A$ with $\tilde{v}_h = v_h$ and doctors' valuations chosen such that $\tilde{v}_{d_1}(x) - \tilde{v}_{d_1}(x') = \tilde{v}_{d_2}(y) - \tilde{v}_{d_2}(y')$, so that v_{-h} exhibits common marginal term valuations, and the inequalities in (2) hold. Such a profile exists because $v_h(x) - v_h(x') > v_h(y) - v_h(y')$ generates the required ordering of surpluses. The resulting equilibria coincide with those in Case 1, and again $u_{d_2}(\hat{X}', \hat{s}') > u_{d_2}(\hat{X}, \hat{s})$.

By relabeling doctors and terms, all symmetric cases follow by the same argument and are omitted. ■

B.8 Proof of Proposition 5

Proof. (i) Consider an allocation with salaries $(X, s) \in \mathcal{E}(\pi_s(T))$ such that $X \in \mathcal{F}(\pi_s(T'))$. Suppose, for contradiction, that $(X, s) \notin \mathcal{E}(\pi_s(T'))$. Since $(X, s) \in \mathcal{E}(\pi_s(T))$, we know that X maximizes total surplus. Hence, because $X \in \mathcal{F}(\pi_s(T'))$ and $T' \subset T$, it follows that in every competitive equilibrium in $\mathcal{E}(\pi_s(T'))$, the allocation must be X (Shapley and Shubik 1971). It remains to show that the vector of salaries s, together with X, constitutes a competitive equilibrium in $\mathcal{E}(\pi_s(T'))$. Since $(X, s) \in \mathcal{E}(\pi_s(T))$, there does not exist any feasible allocation $Y \in \mathcal{F}(\pi_s(T'))$, and a vector of salaries s' such that $Y_h \in \Phi_h(s')$ and $Y_d \in \Phi_d(s')$ for each $d \in D(Y_h)$, with $p_h(Y, s') > p_h(X, s)$ or $u_d(Y, s') > u_d(X, s)$, for some $d \in D(Y_h)$. Therefore, such an allocation and vector of salaries cannot exist in $\pi_s(T')$ either. Moreover, since (X, s) is a competitive equilibrium, no doctor d satisfies $u_d(\varnothing, s_\varnothing) > u_d(X, s)$, and no hospital h satisfies $p_h(\varnothing, s_\varnothing) > p_h(X, s)$. Hence, $(X, s) \in \mathcal{E}(\pi_s(T'))$.

(ii) Suppose, for contradiction, that there exists $(X, s) \in \mathcal{E}(\pi_s(T')) \cap \mathcal{E}(\pi_s(T''))$ and $(X, s) \notin \mathcal{E}(\pi_s(T' \cup T''))$. We first show that X remains the allocation used in a competitive equilibrium of $\pi_s(T' \cup T'')$. For each doctor $d \in D$, because (X, s) is a competitive equilibrium in both $\pi_s(T')$ and $\pi_s(T'')$, we have $X_d \in \Phi_d(s)$ in each sub-problem. It follows immediately that $X_d \in \Phi_d(s)$ in $\pi_s(T' \cup T'')$, since any contract available to doctor d in $\pi_s(T' \cup T'')$ is already available in one of the two sub-problems. Similarly, for each hospital $h \in H$, since $X_h \in \Phi_h(s)$ in both $\pi_s(T')$ and $\pi_s(T'')$, and hospital demand satisfies the gross substitutes condition, enlarging the sets of terms offered by hospitals from T' and T'' to $T' \cup T''$ while keeping salaries fixed does not render any contract in X_h to cease to be demanded. Hence, $X_h \in \Phi_h(s)$ in $\pi_s(T' \cup T'')$. Using the same reasoning as in part (i), there does not exist any feasible allocation $Y \in \mathcal{F}(\pi_s(T' \cup T''))$, and a vector of salaries s' such that $Y_h \in \Phi_h(s')$ and $Y_d \in \Phi_d(s')$ for each $d \in D(Y_h)$, with $p_h(Y, s') > p_h(X, s)$ or $u_d(Y, s') > u_d(X, s)$, for some $d \in D(Y_h)$. Moreover, since (X, s) is a competitive equilibrium, no doctor d satisfies $u_d(\varnothing, s_\varnothing) > u_d(X, s)$, and no hospital h satisfies $p_h(\varnothing, s_\varnothing) > p_h(X, s)$. Hence, $(X, s) \in \mathcal{E}(\pi_s(T' \cup T''))$. ■

B.9 Proof of Proposition 6

Proof. Consider a problem with salaries $\pi_s(T)$, and suppose there exists $T' \subset T$ such that $u_d(\hat{X}', \hat{s}') > u_d(\hat{X}, \hat{s})$ for each $d \in D$. Since $(\hat{X}, \hat{s})$ is a competitive equilibrium, for each $d \in D$, $u_d(\hat{X}', \hat{s}') > 0$. This implies that for each $d \in D$, $\hat{X}'_d \neq \{\varnothing\}$, and therefore $|D| \leq \sum_{h \in H} q_h$. The following lemma establishes that in this case, at least one hospital earns zero profit at the doctor-optimal competitive equilibrium.

Lemma 3. For any problem with salaries $\pi_s(T) \in \Pi_s$, if $|D| \leq \sum_{h \in H} q_h$, then there exists at least one hospital $h \in H$ such that $p_h(\hat{X}, \hat{s}) = 0$.

Proof. Suppose, for contradiction, that $p_h(\hat{X}, \hat{s}) > 0$ for every $h \in H$. Then there exists $\varepsilon > 0$ sufficiently small such that $p_h(\hat{X}, \hat{s}) \geq \varepsilon$ for every $h \in H$. Since revenue functions are semicontinuous and utility functions are continuous, and hospitals' demands satisfy the gross substitutes condition, the set of competitive equilibrium utility vectors forms a lattice (Kelso and Crawford 1982). We can define a vector of salaries $s^\varepsilon > \hat{s}$ such that $\sum_{x \in \hat{X}} (s^\varepsilon_x - \hat{s}_x) \leq \varepsilon$. It follows that $u_d(\hat{X}, \hat{s}^\varepsilon) \geq u_d(\hat{X}, \hat{s})$ for every $d \in D$, with strict inequality for at least one doctor, since $s^\varepsilon > \hat{s}$. Moreover, since the total salary increase under $\hat{X}$ is at most ε, it follows that $p_h(\hat{X}, s^\varepsilon) \geq 0$ for every $h \in H$. By continuity of utility functions and semicontinuity of revenue functions, it follows that $\hat{X}_a \in \Phi_a(s^\varepsilon)$ for every $a \in D \cup H$. Hence, $(\hat{X}, s^\varepsilon)$ is a competitive equilibrium. Since at least one component of s^ε is strictly greater than the corresponding component of $\hat{s}$, at least one doctor is strictly better off under $(\hat{X}, s^\varepsilon)$, while every other doctor is weakly better off. This contradicts the doctor-optimality of $(\hat{X}, \hat{s})$. ■

Therefore, there must exist at least one hospital h' with $p_h(\hat{X}'_h, \hat{s}') = 0$. In $(\hat{X}', \hat{s}')$, let d' denote a doctor employed by h' such that $u_{d'}(\hat{X}', \hat{s}') = v_{h'}(\hat{X}') - v_h(\hat{X}' \setminus \hat{X}'_{d'}) + v_{d'}(\hat{X}'_{d'})$, that is d' obtains the entire surplus generated by $\hat{X}_{d'}$. Since $\hat{X}' \in \mathcal{F}(\pi_s(T))$, either d' remains employed by h' in $\hat{X}$, in which case the maximization of total surplus implies $u_{d'}(\hat{X}, \hat{s}) \geq u_{d'}(\hat{X}', \hat{s}')$, or d' is employed by another hospital in $\hat{X}$, and $u_{d'}(\hat{X}, \hat{s}) \geq u_{d'}(\hat{X}', \hat{s}')$. In both cases, we obtain a contradiction. ■

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