Supplement to "Information Design in Smooth Games"

ALEX SMOLIN
Department of Economics, Toulouse School of Economics

TAKURO YAMASHITA
Institute of Social and Economic Research, Osaka University

In this appendix, we apply the certification approach to a game with non-quadratic payoffs.

Imagine a designer who can inform a representative retailer and a representative consumer about a new two-sided application. The application is either high quality ($\omega = 1$) or low quality ($\omega = 0$).

The retailer chooses a search intensity $a_1 \in [0,1]$ in the application, and the consumer chooses $a_2 \in [0,1]$. Each player i incurs the cost

$$c(a_i) = \begin{cases} \hat{a}^3 a_i, & a_i \in [0, \hat{a}), \\ \frac{a_i^4}{4} + \frac{3\hat{a}^4}{4}, & a_i \in [\hat{a}, 1]. \end{cases}$$

where $\hat{a} = 3/4$. Thus c is smooth and convex, and $\hat{a}$ minimizes the average-cost function.

The players' payoffs are

$$u_i(a, \omega) = \omega\, a_i a_{-i} - c(a_i).$$

A higher search intensity a_i raises player i's chance of finding a match, and this benefit is greater when the state is high and when the other player searches more intensively. Hence the game exhibits strategic complementarities.

The designer aims to raise overall search intensity to boost engagement and advertising revenue. Her payoff is

$$v(a, \omega) = a_1 + a_2.$$

Public Information First, suppose the platform sends a public signal, identical for both players. The optimal signal can then be derived via the standard concavification procedure. Let μ denote the common belief that $\omega = 1$. Given this belief, the players play either (i) $a_1 = a_2 = a^*(\mu) \geq \hat{a}$, where

$$\mu\, a^*(\mu) = \big(a^*(\mu)\big)^3,$$

so $a^*(\mu) = \sqrt{\mu} \geq \hat{a}$; or (ii) $a_1 = a_2 = 0$. Consequently, the designer's indirect utility equals $2\sqrt{\mu}$ whenever $\sqrt{\mu} \geq \hat{a}$ and 0 otherwise.

Hence, if the prior satisfies $\mu_0 \geq \hat{a}^2$, no disclosure is optimal. If $\mu_0 < \hat{a}^2$, the optimal public signal splits posteriors between 0 and $\hat{a}^2$; concretely, (i) when $\omega = 0$, the posterior is either 0 or $\hat{a}^2$, leading to actions $(a_1, a_2) = (0,0)$ or $(\hat{a}, \hat{a})$; (ii) when $\omega = 1$, the posterior is always $\hat{a}^2$, so $(a_1, a_2) = (\hat{a}, \hat{a})$.

Optimal Information Now, we show that the above public information is optimal even within the class of all private information structures.

The dual payoff is

$$w(a,\omega) = a_1 + a_2 - \lambda_1(a_1)(\omega a_2 - c'(a_1)) - \lambda_2(a_2)(\omega a_1 - c'(a_2)).$$

Posit the following certificate:

$$\lambda_i(a_i) = \begin{cases} -\frac{a_i}{\hat{a}^3}, \text{ if } a_i < \hat{a}, \\ -\frac{1}{a_i^2}, \text{ if } a_i \geq \hat{a}, \end{cases}$$

so that

$$w(a,\omega) = \begin{cases} \omega(\frac{a_2}{a_1^2} + \frac{a_1}{a_2^2}), & \text{if } a_1, a_2 \geq \hat{a}, \\ \omega(\frac{a_2}{a_1^2} + \frac{a_1 a_2}{\hat{a}^3}), & \text{if } a_1 \geq \hat{a} > a_2, \\ \omega(\frac{a_1}{a_2^2} + \frac{a_1 a_2}{\hat{a}^3}), & \text{if } a_2 \geq \hat{a} > a_1, \\ \omega(\frac{2a_1 a_2}{\hat{a}^3}), & \text{if } a_1, a_2 < \hat{a}. \end{cases}$$

From the perspective of the dual agent, if $\omega = 0$, then any a is optimal, including $a = (0,0)$ and $(\hat{a}, \hat{a})$. If $\omega = 1$, we can focus on the case $a_1, a_2 \geq \hat{a}$. Because $w(a,\omega)$ is convex on this domain, its maximum must occur at one of the corner points $a = (\hat{a}, \hat{a})$, $(\hat{a}, 1)$, $(1, \hat{a})$, or $(1,1)$. A straightforward calculation shows that $(\hat{a}, \hat{a})$ is optimal whenever $1 - \hat{a} - \hat{a}^2 \leq 0$, which indeed holds for $\hat{a} = 3/4$. The result follows.